



# Geometry Processing and Machine Learning for Modeling and Transmission of Complex 3D Scenes

Pierre Alliez

Project-team T I T A N E

Geometric Modeling of 3D Environments

<https://team.inria.fr/titane/>

# Outline

## CORESA topics

- Progressive compression
- Inter-surface mapping
- Higher-order meshes

## Recent and outlook

- Cognitive 3D models
- Physics-informed modeling

# CORESA Topics

# Progressive Compression

Joint work with Cédric Portaneri, Michael Hemmer,  
Lukos Birklein and Elmar Schoemer



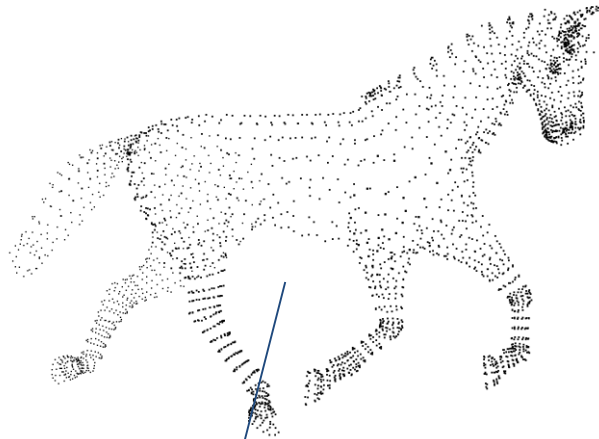
# Textured Mesh



# Textured Mesh Components

## Geometry

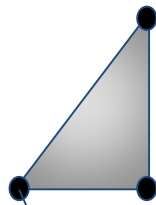
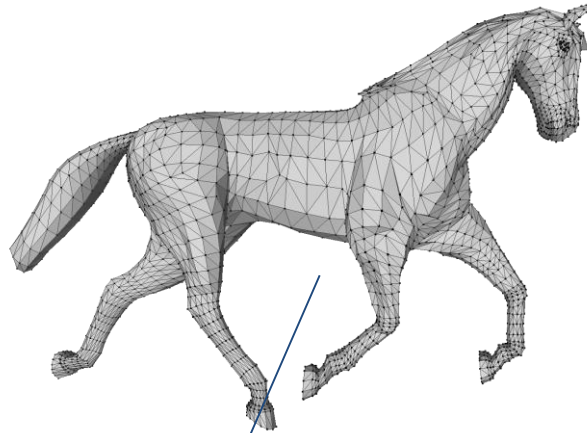
Location of vertices



3D coordinates

## Connectivity

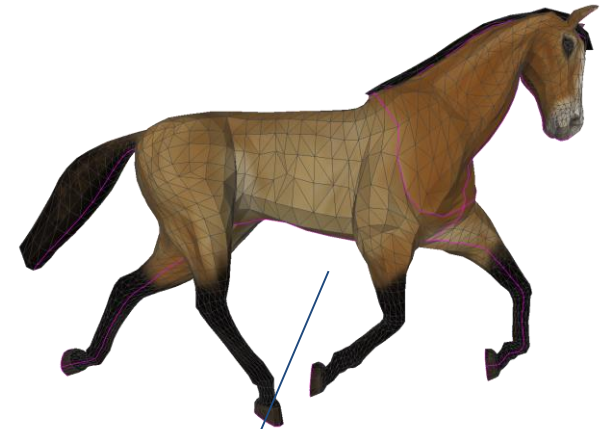
How vertices are connected



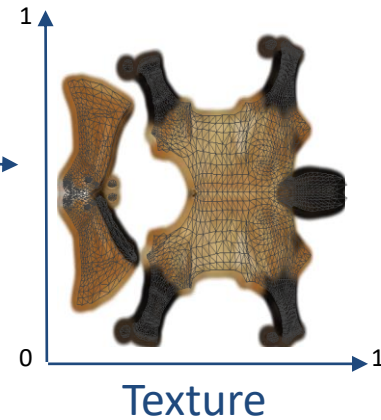
Vertex indices

## Texture Mapping

How the texture is applied

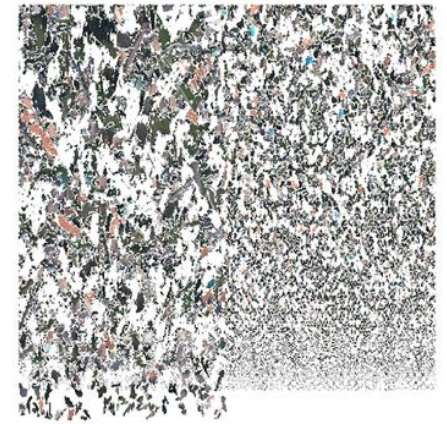
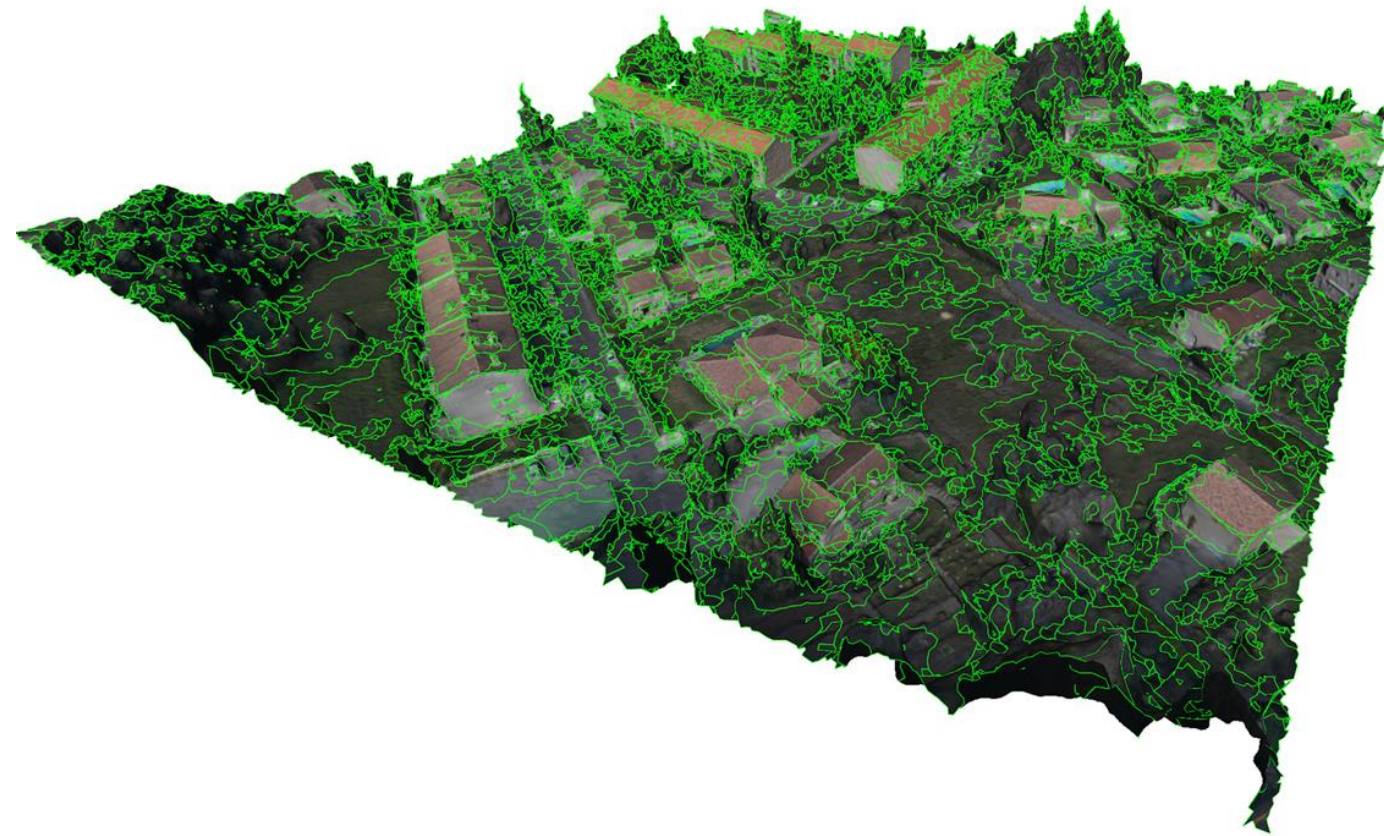


2D coordinates (uv)



# Context

Increasingly common: Automatically generated texture seams



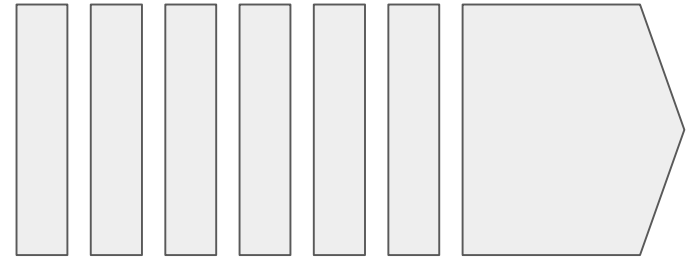
# Progressive Compression



3D Model



Compression



Stream of refinements



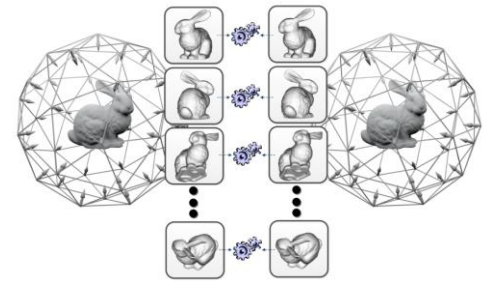
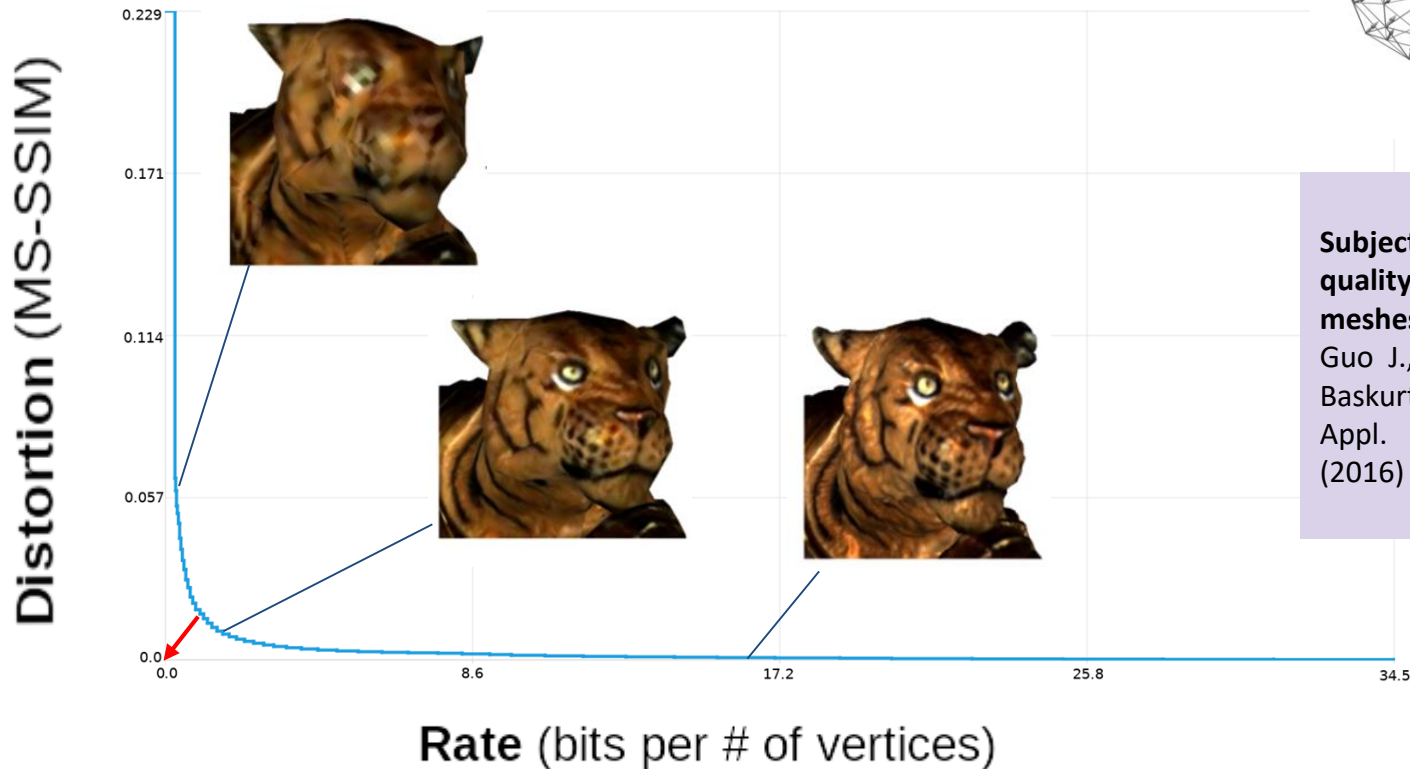
1st Level of Detail  
(single-rate encoded)





# Progressive Compression

Goal: optimize rate-distortion trade-off



**Subjective and objective visual quality assessment of textured 3D meshes.**

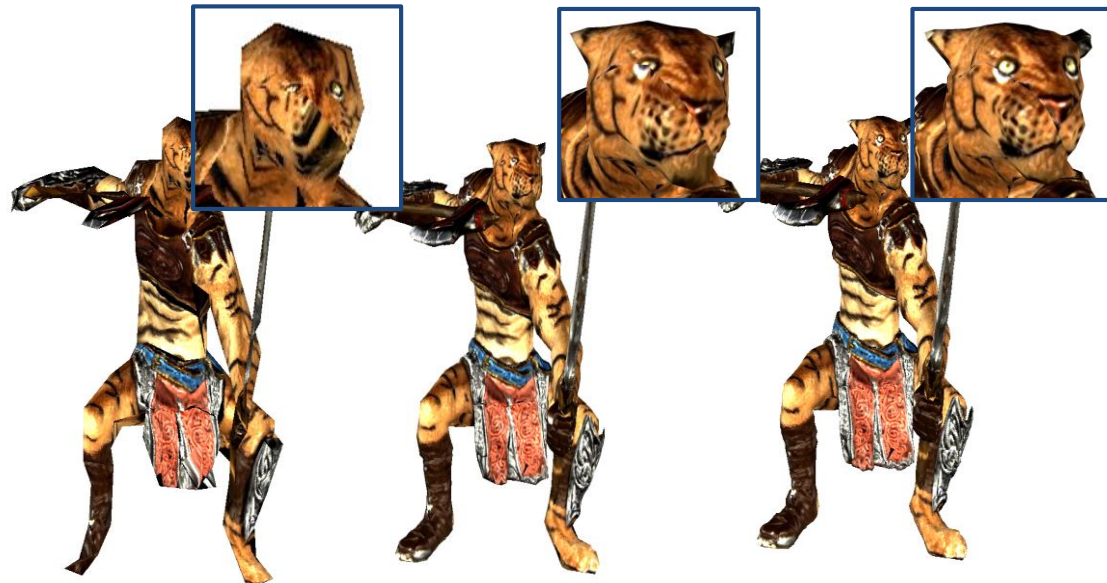
Guo J., Vidal V., Cheng I., Basu A., Baskurt A., Lavoué G. ACM Trans. Appl. Percept. 14(2), 11:1–11:20 (2016)

# Previous Work

**Progressive Streaming of Textured 3D Models in a Web Browser.** Lavoué et al. In Proceedings of the 20th ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games, 2016



**Progressive Compression of Arbitrary Textured Meshes.** Caillaud, Vidal, Dupont, Lavoué. Computer Graphics Forum (Pacific Graphics), 2016.



# Global Quantization



Input mesh



14 bits quantization  
on geometry

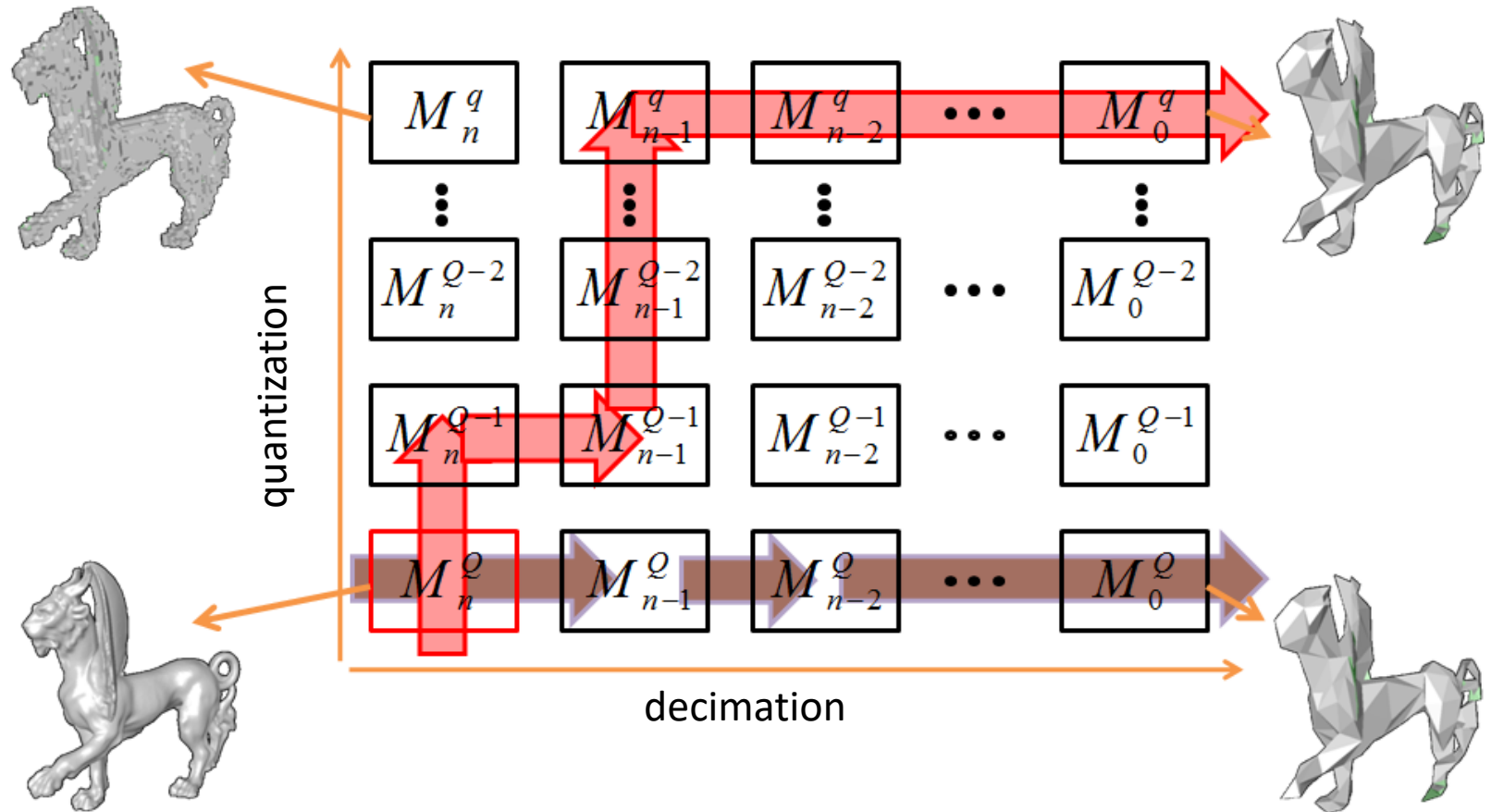


6 bits quantization  
on geometry

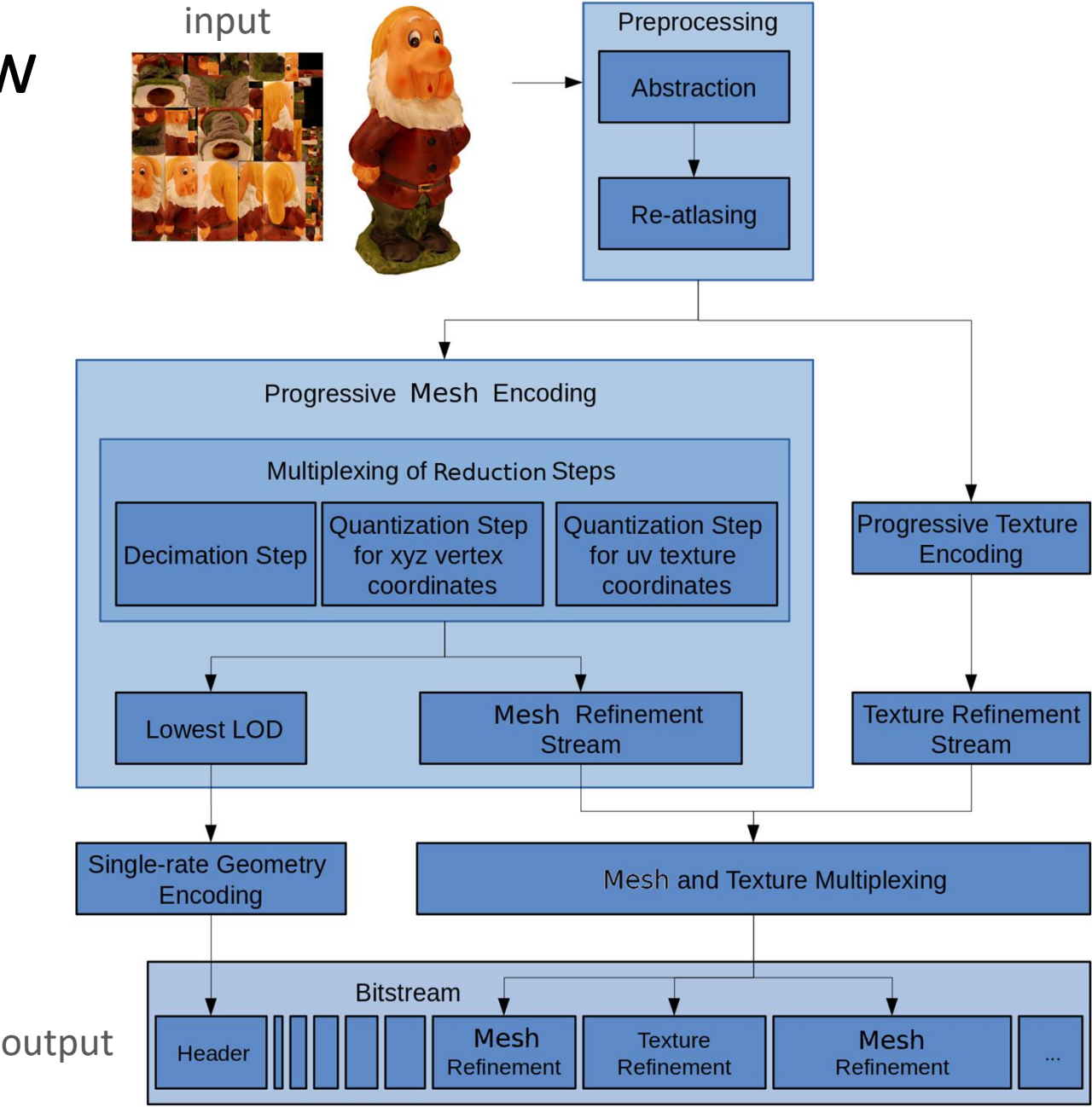


6 bits quantization  
on texture mapping

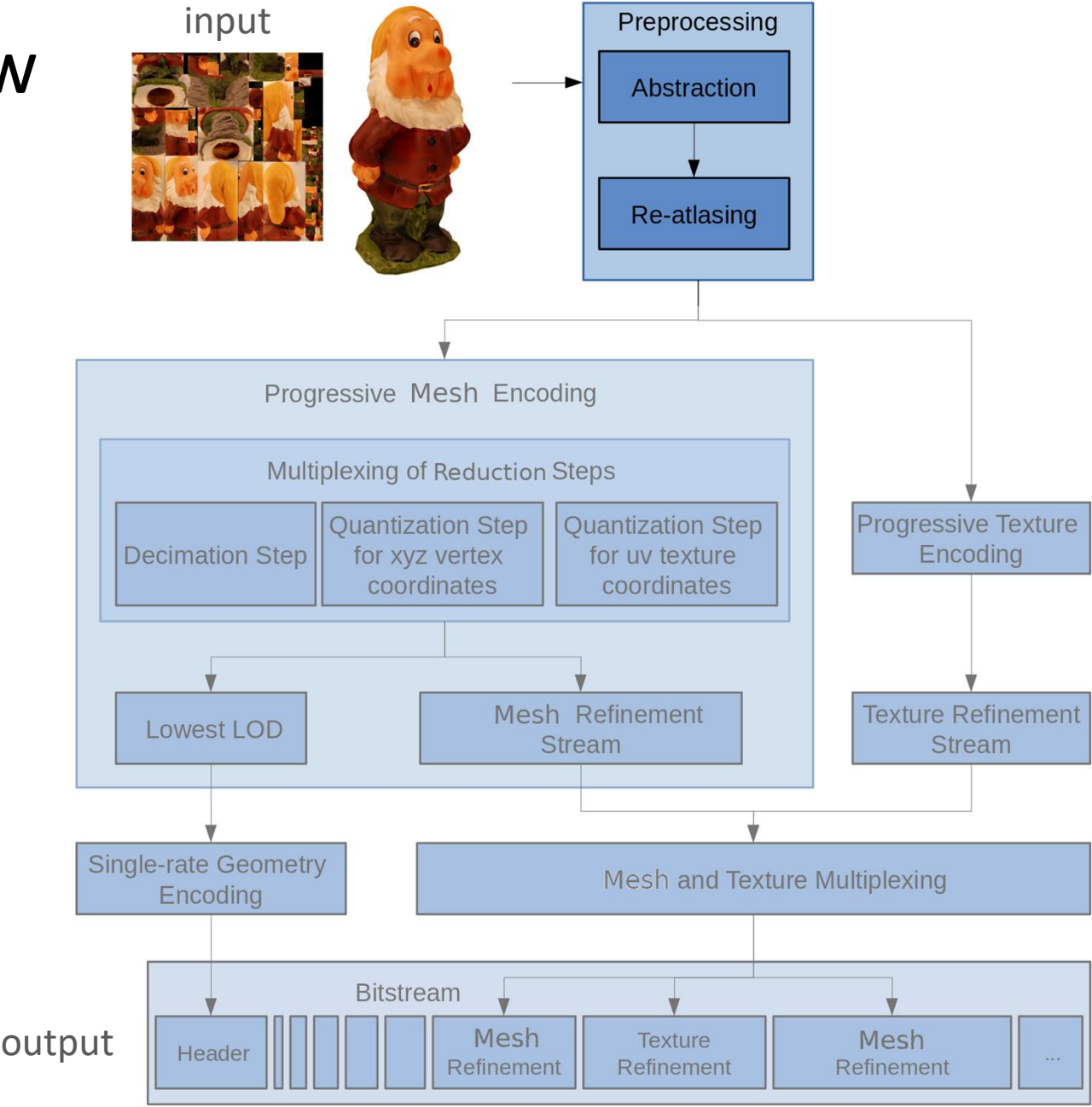
# Adaptive Quantization [Lee, Lavoué, Dupont 09]



# Overview

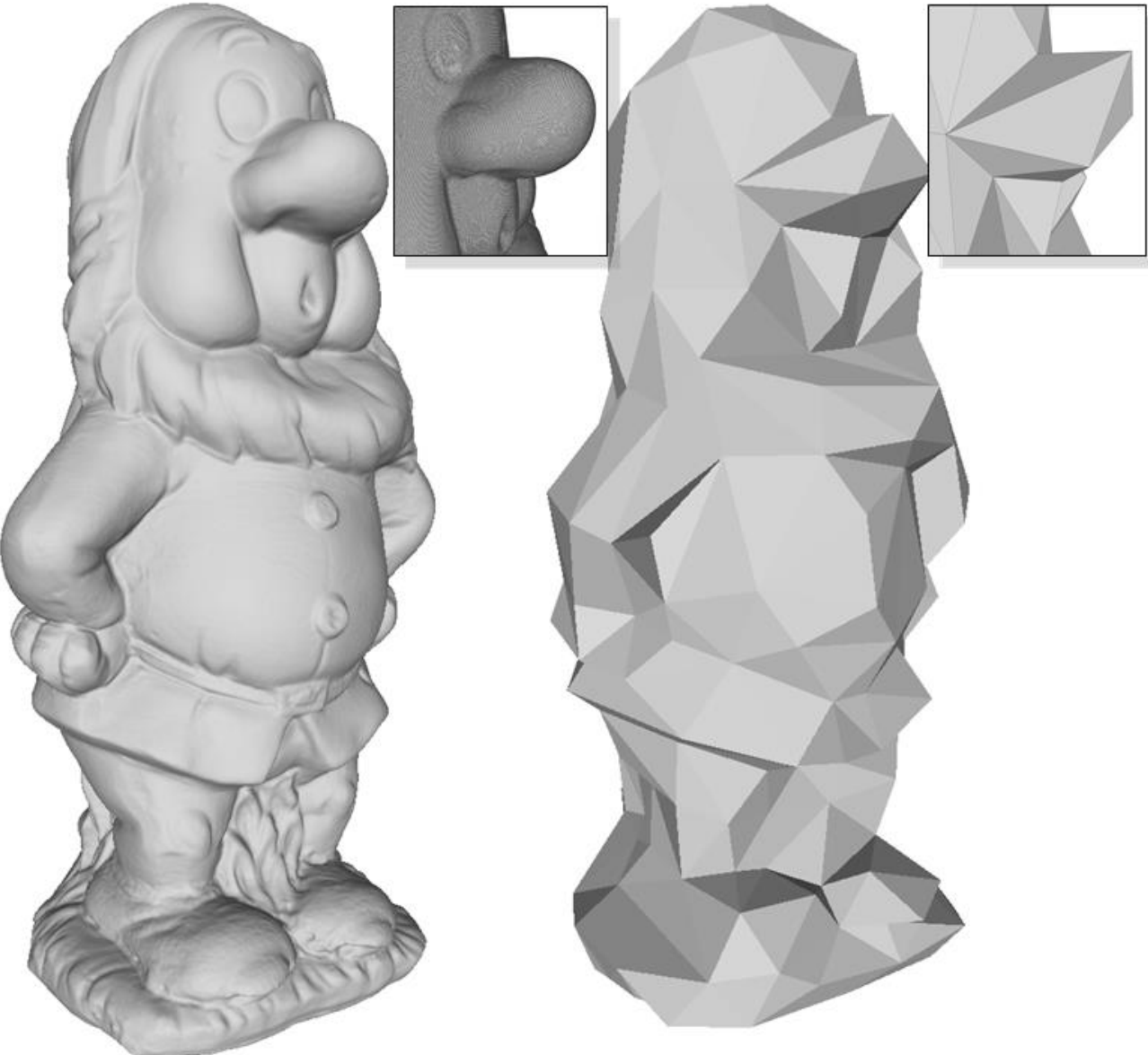


# Overview



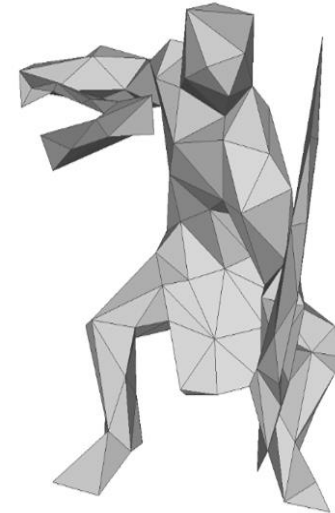
# Preprocessing

## Abstraction



# Preprocessing

## Re-atlasing



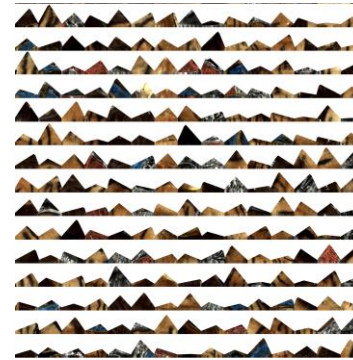
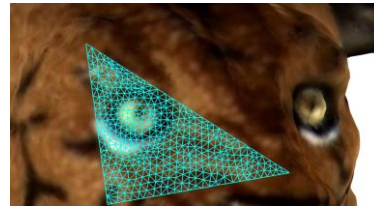
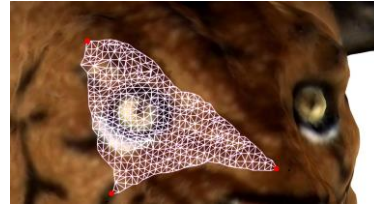
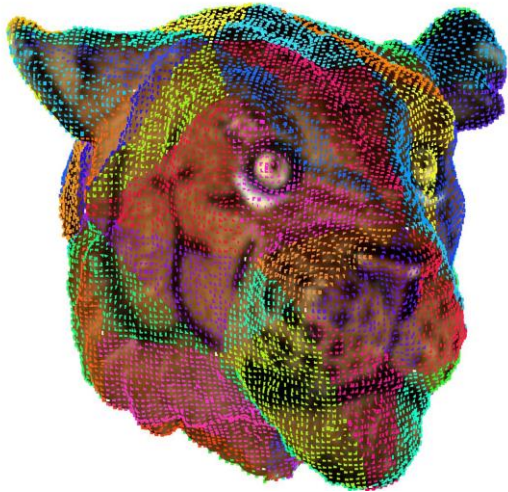


# Decimation toward Abstraction

Seam-preserving simplification



# Re-atlasing Steps

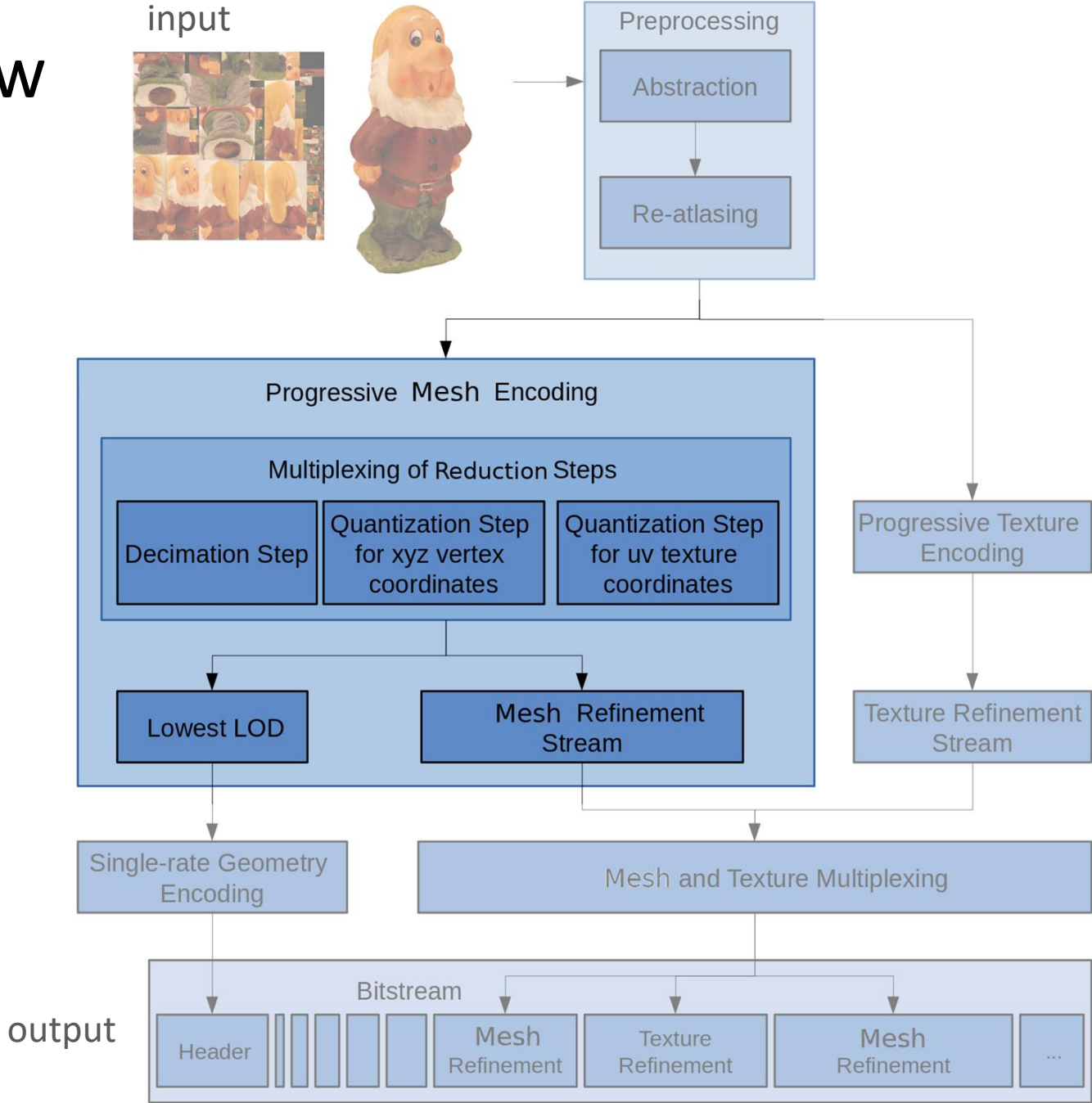


1. Cluster input triangles onto abstraction triangle

2. Form a texture patch by planar parameterization

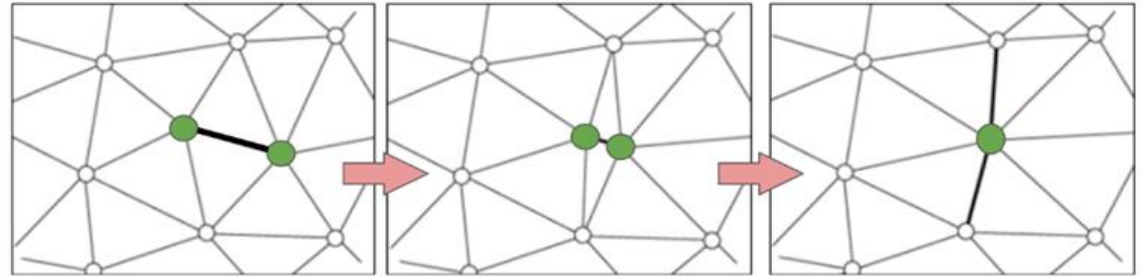
3. Pack the texture patches.  
Glue texture seams and save them as virtual seams

# Overview

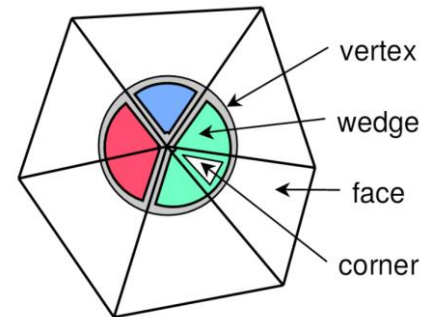
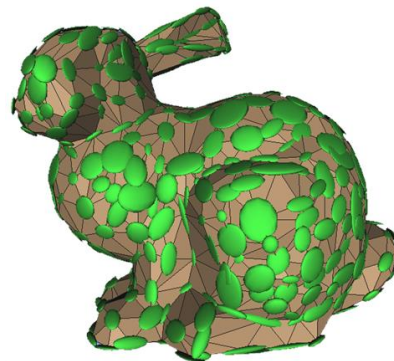
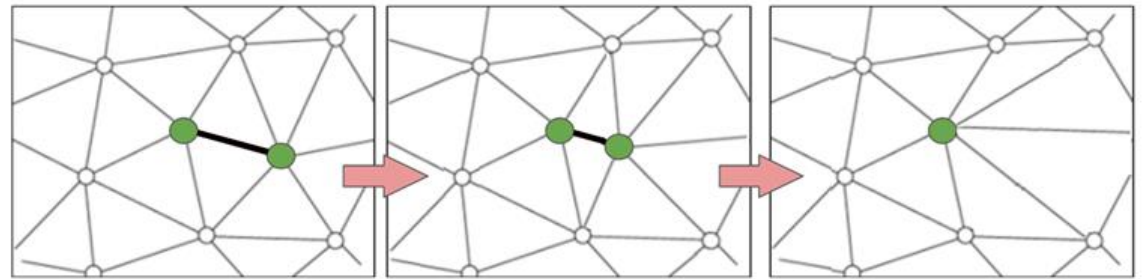


# Decimation Operators

Full-edge collapse



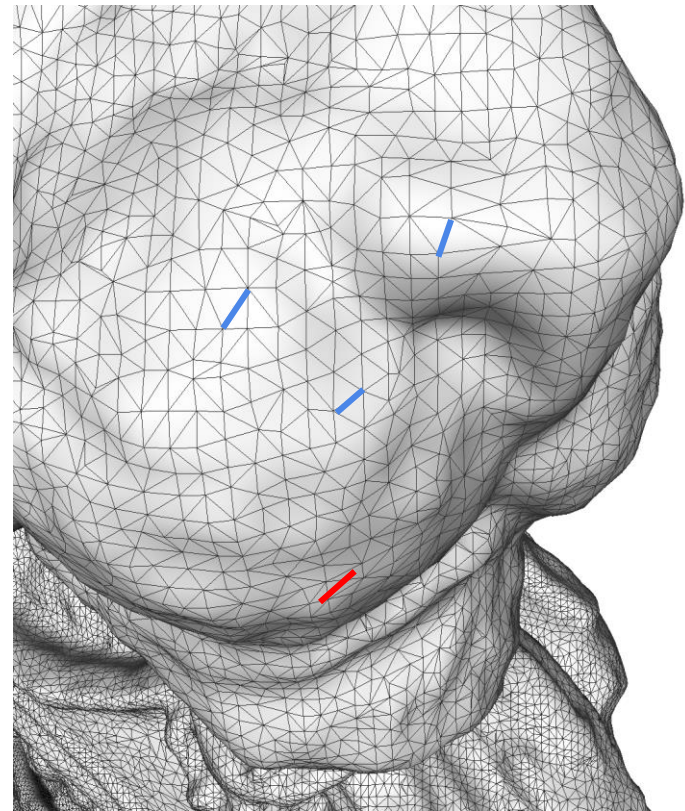
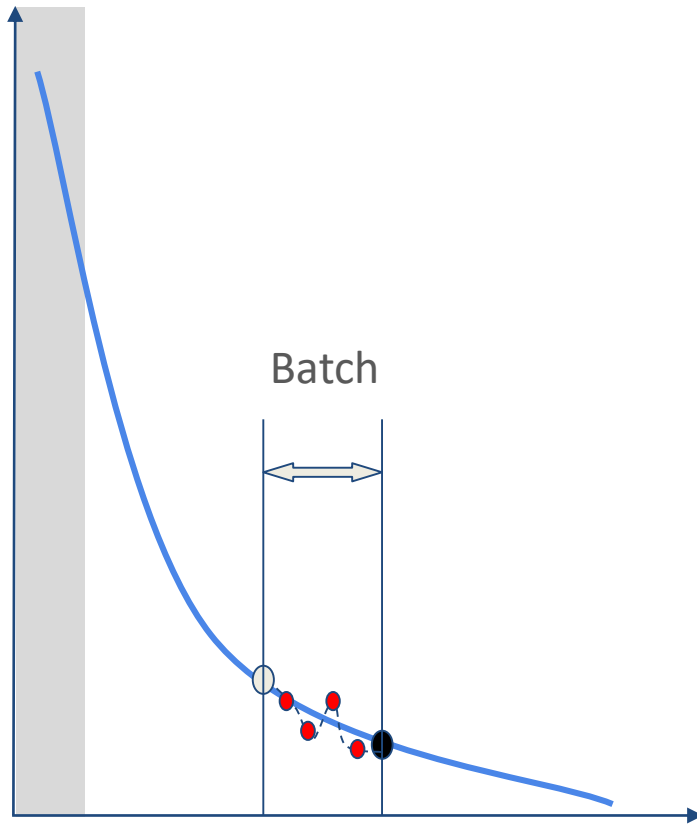
Half-edge collapse  
(cheaper to encode)



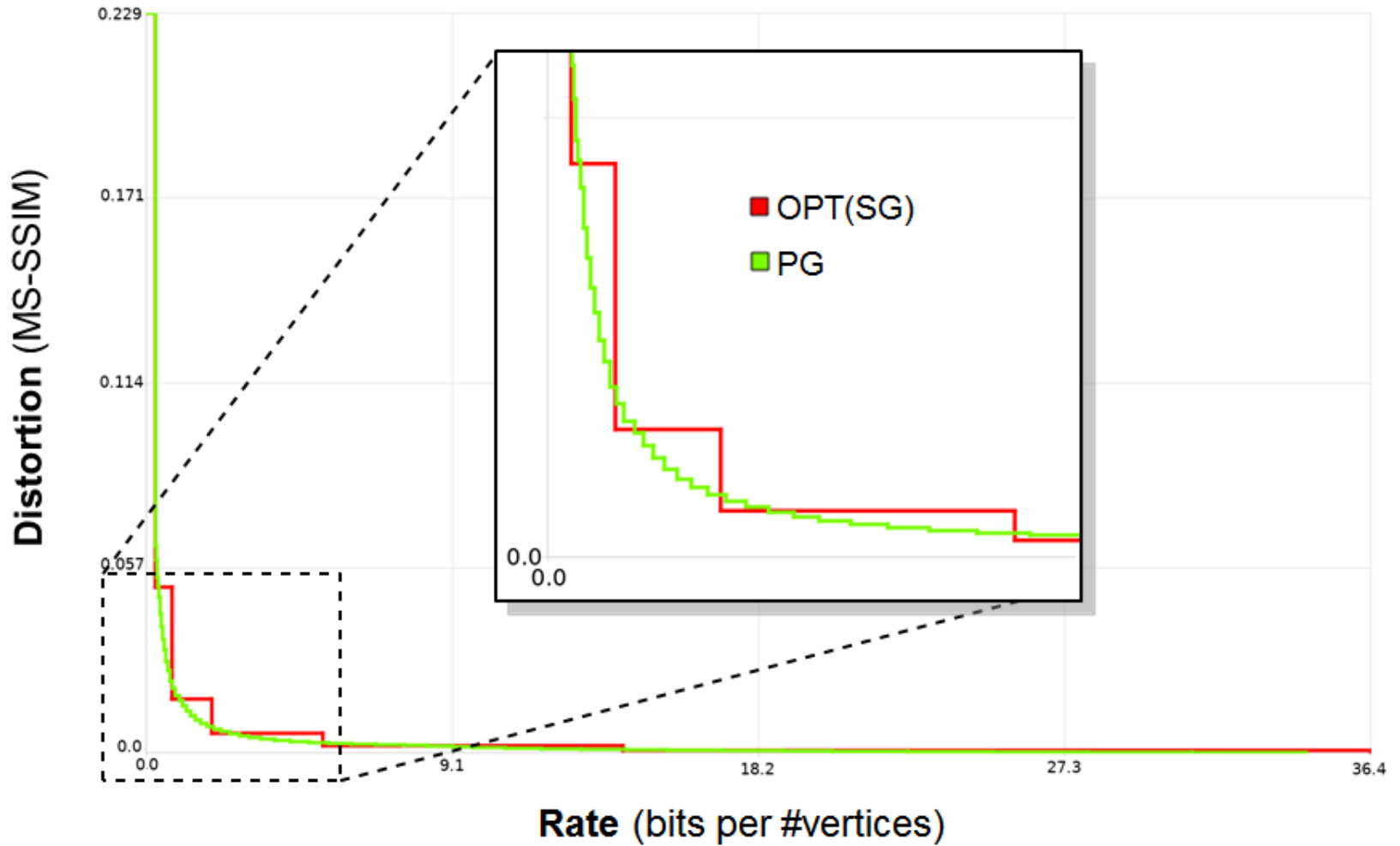
**5D Quadric Error Metric**

# Decimation

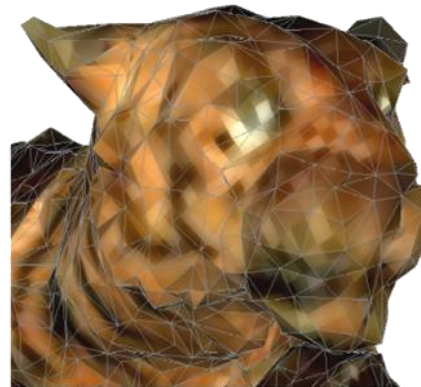
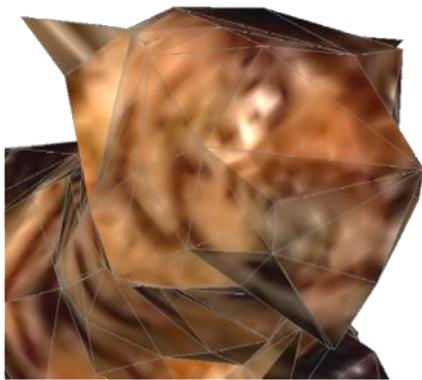
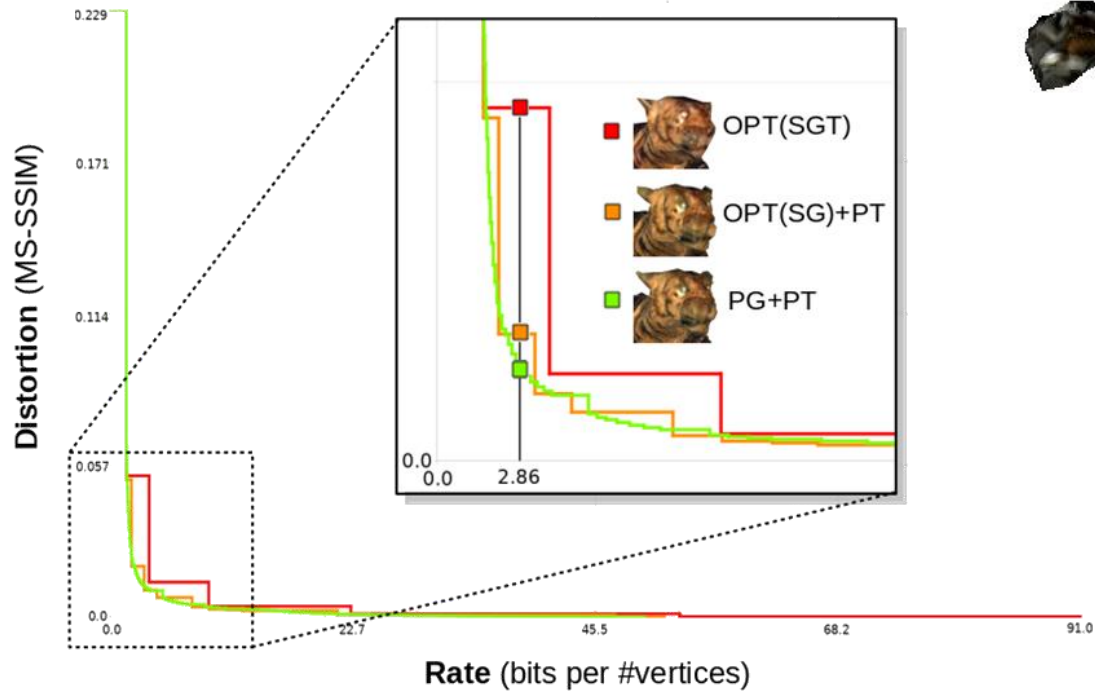
Order of decimation operator



# Results



# Results

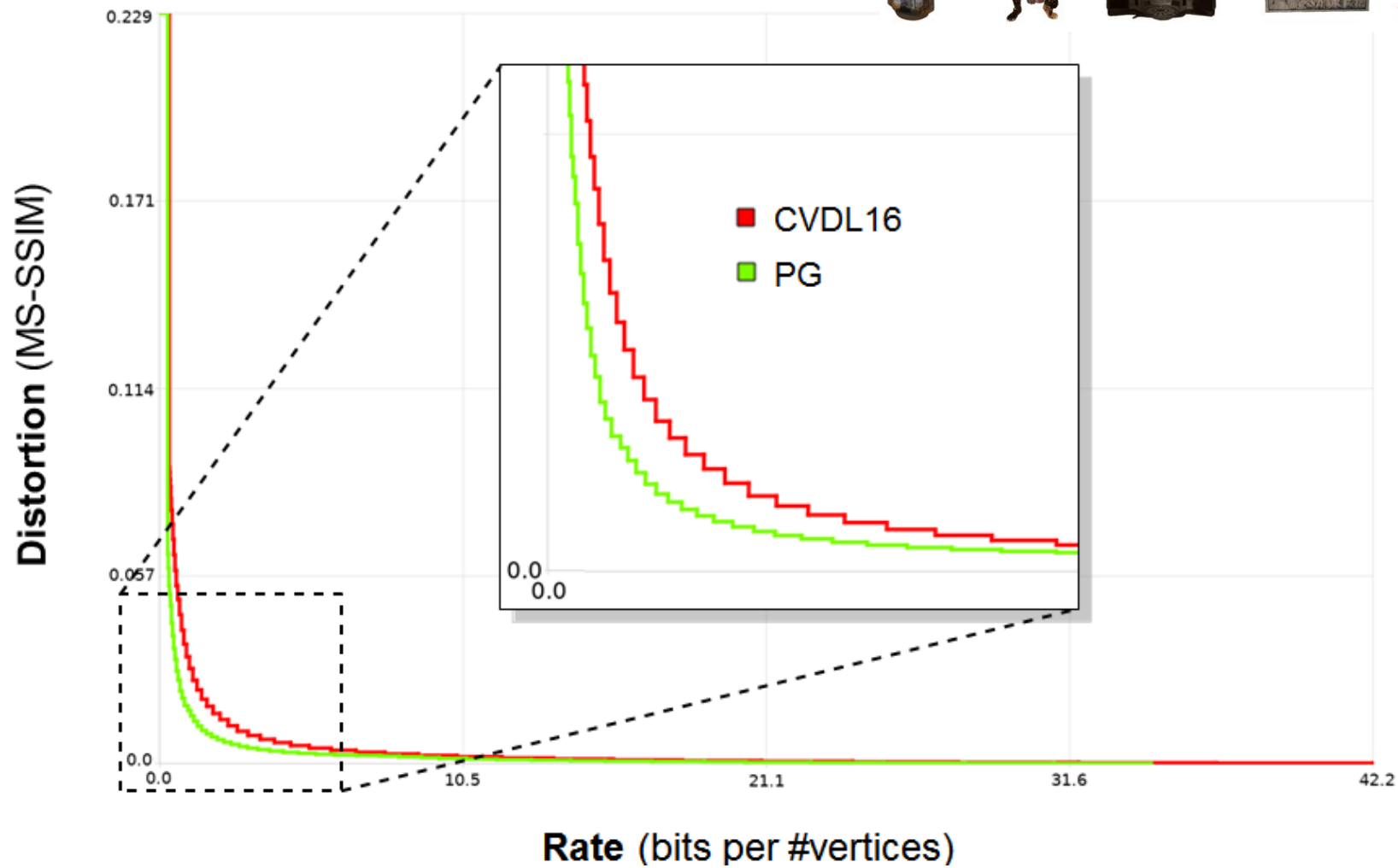


■ OPT(SGT)

■ OPT(SG)+PT

■ PG+PT

# Results





# Inter-surface mapping

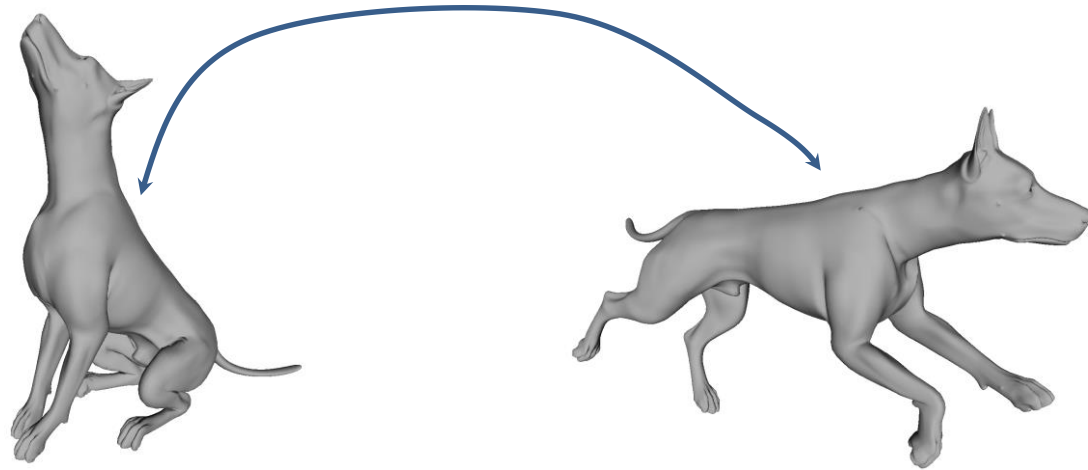
Joint work with **Manish Mandad**,  
David Cohen-Steiner, Leif Kobbelt,  
and Mathieu Desbrun



30 JULY - 3 AUGUST *Los Angeles*  
**SIGGRAPH**2017



# Problem Statement



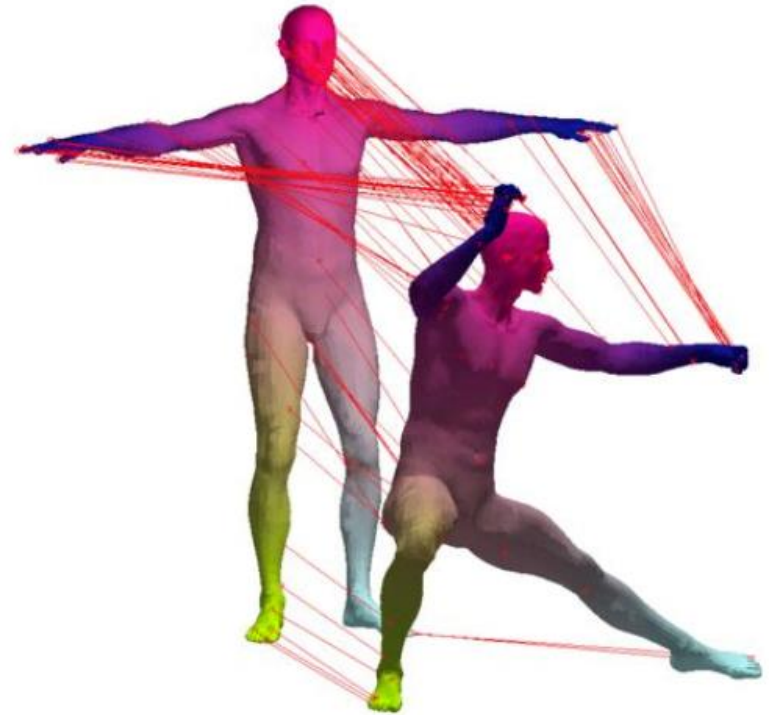
Find mapping between two surfaces

- Bijective
- Some degree of regularity
- Semantically meaningful

# Inter-Surface Mapping

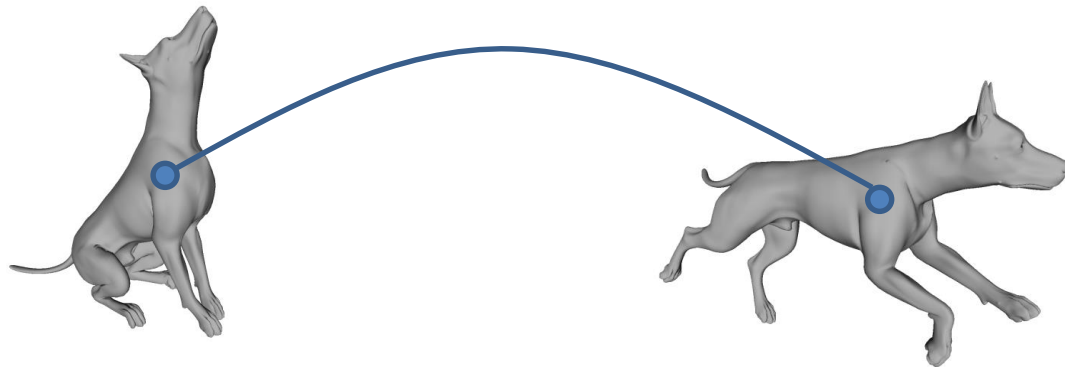
Used for:

- Template fitting
- Detail or attribute transfer
- Blending
- Morphing
- Surface reconstruction
- Remeshing
- ...



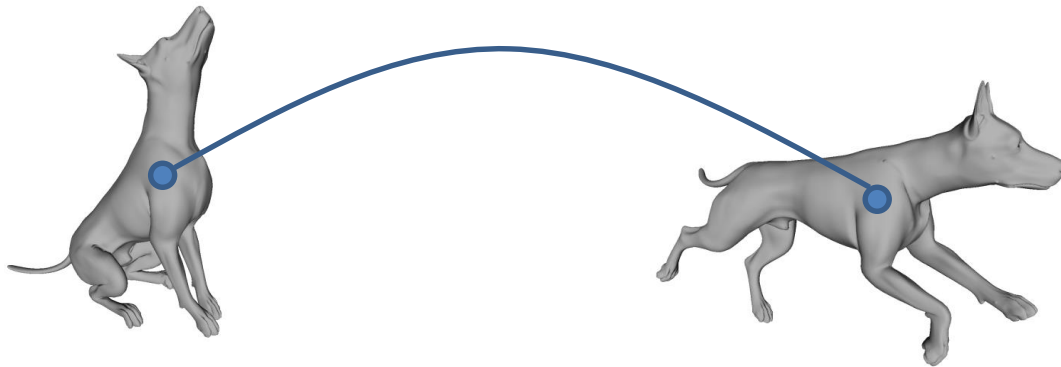
# Homeomorphism?

- Bi-continuous function between topological spaces
- « continuous stretching and bending of the object into a new shape»
- ...
- Neighborhoods map to neighborhoods



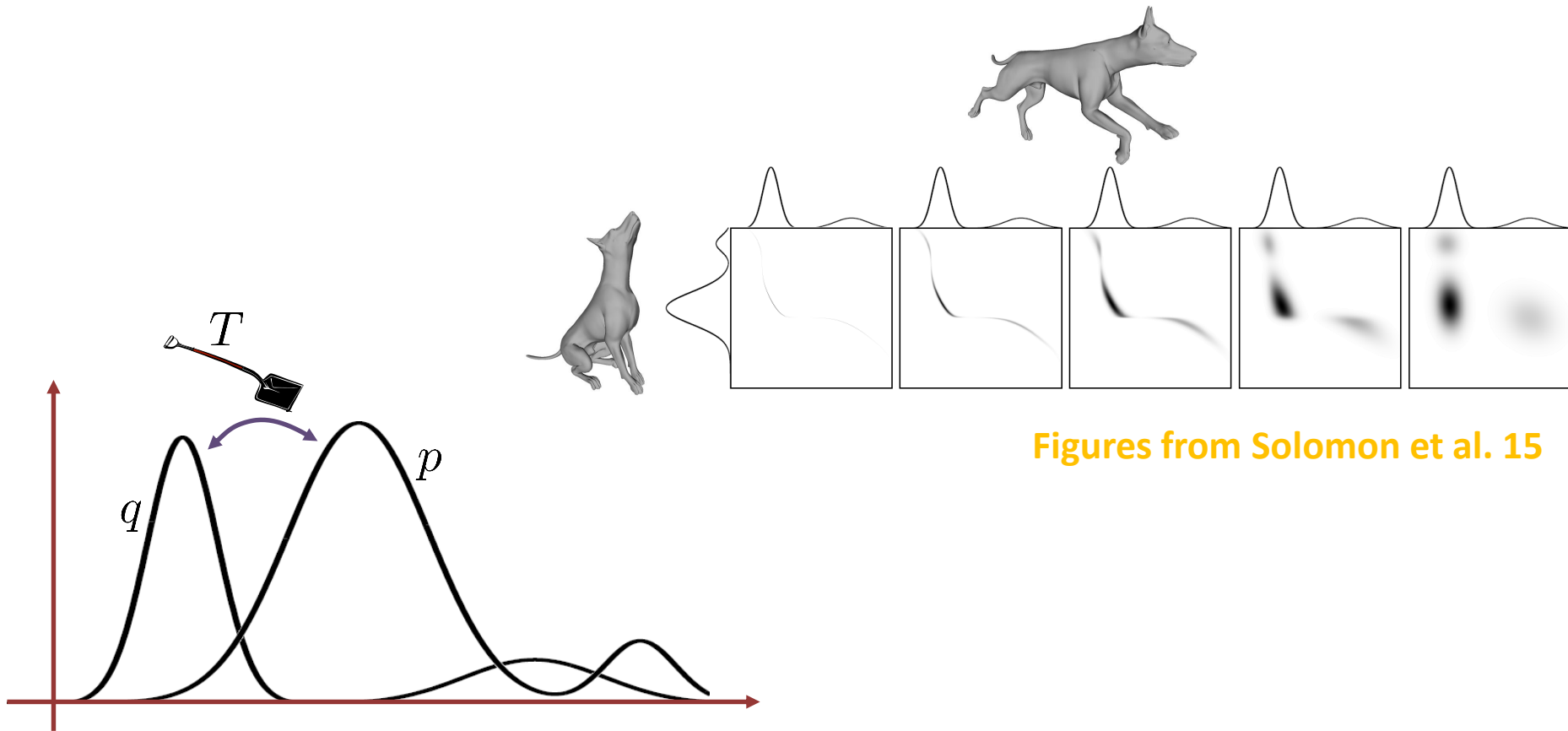
# Homeomorphism?

- *Geodesic neighbourhoods map to small geodesic neighbourhoods*



# Proposal

Formulation based on **Optimal Transportation**





# Formulation

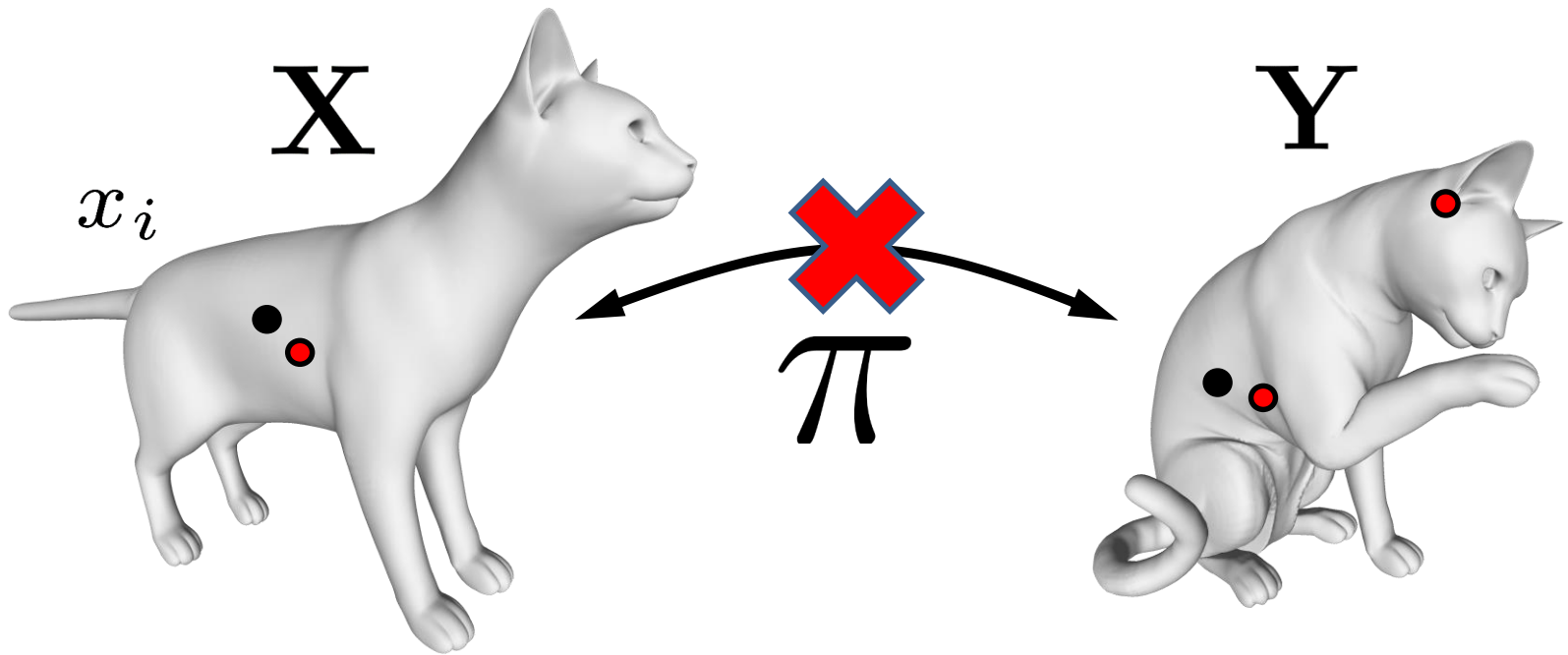
- Minimize variances of images of neighborhoods by the transport plan.
- Penalizes stretching and favors point-to-point homeomorphisms.

weighting function

$$E(\pi) = \int_{\mathbf{X}} \text{var } \pi_{\mathbf{X}} \left( \frac{W_x \mu}{\text{mass}(W_x \mu)} \right) d\mu(x) + \int_{\mathbf{Y}} \text{var } \pi_{\mathbf{Y}} \left( \frac{W_y \nu}{\text{mass}(W_y \nu)} \right) d\nu(y)$$

The diagram illustrates the formulation. It shows a sequence of points  $X$  (teal) and  $Y$  (orange) with arrows representing a transport plan. A weighting function  $W_x \mu$  is shown as a curve over  $X$ . A blue arc connects two dog images, representing a point-to-point homeomorphism.

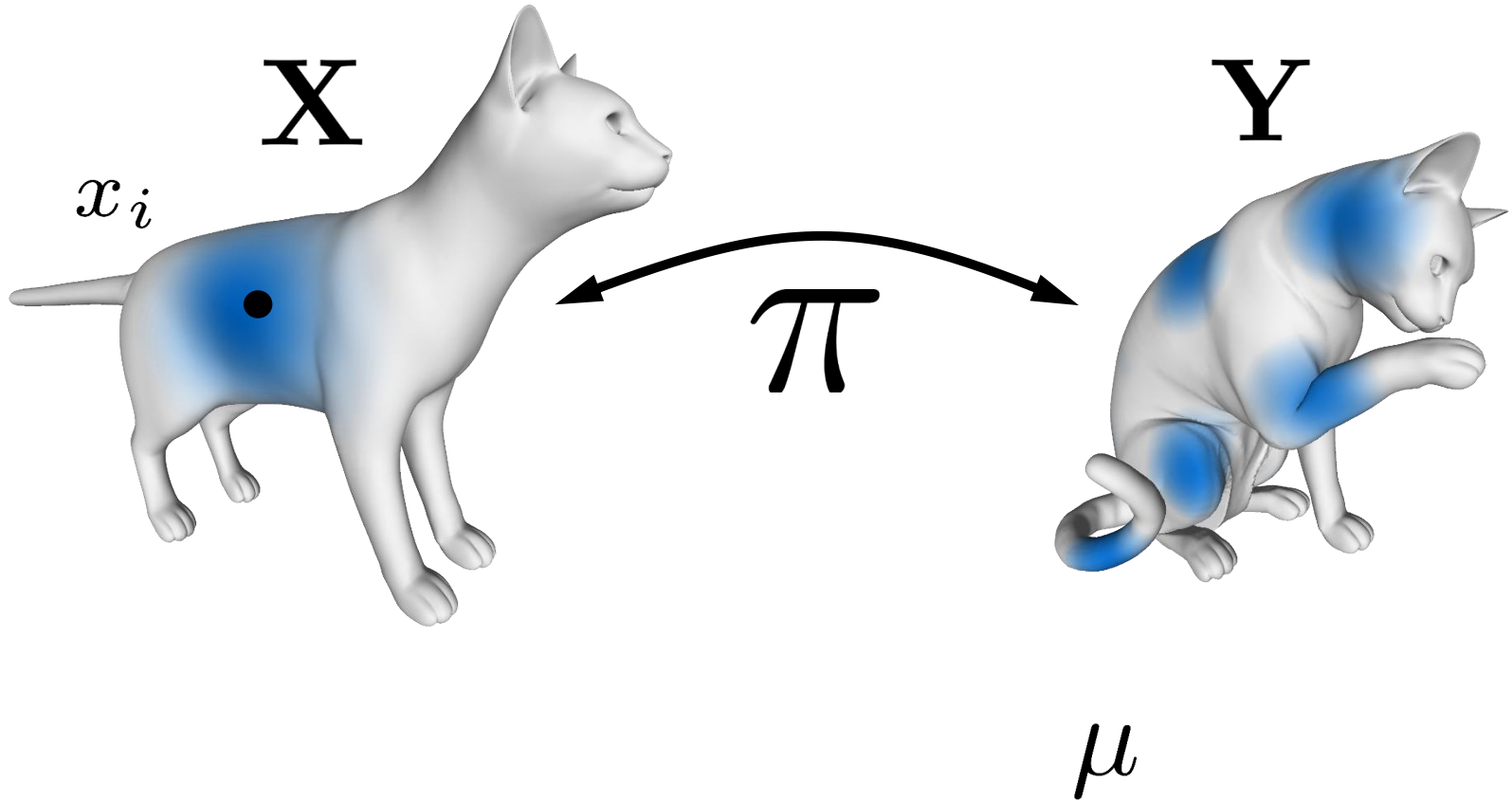
# Continuity



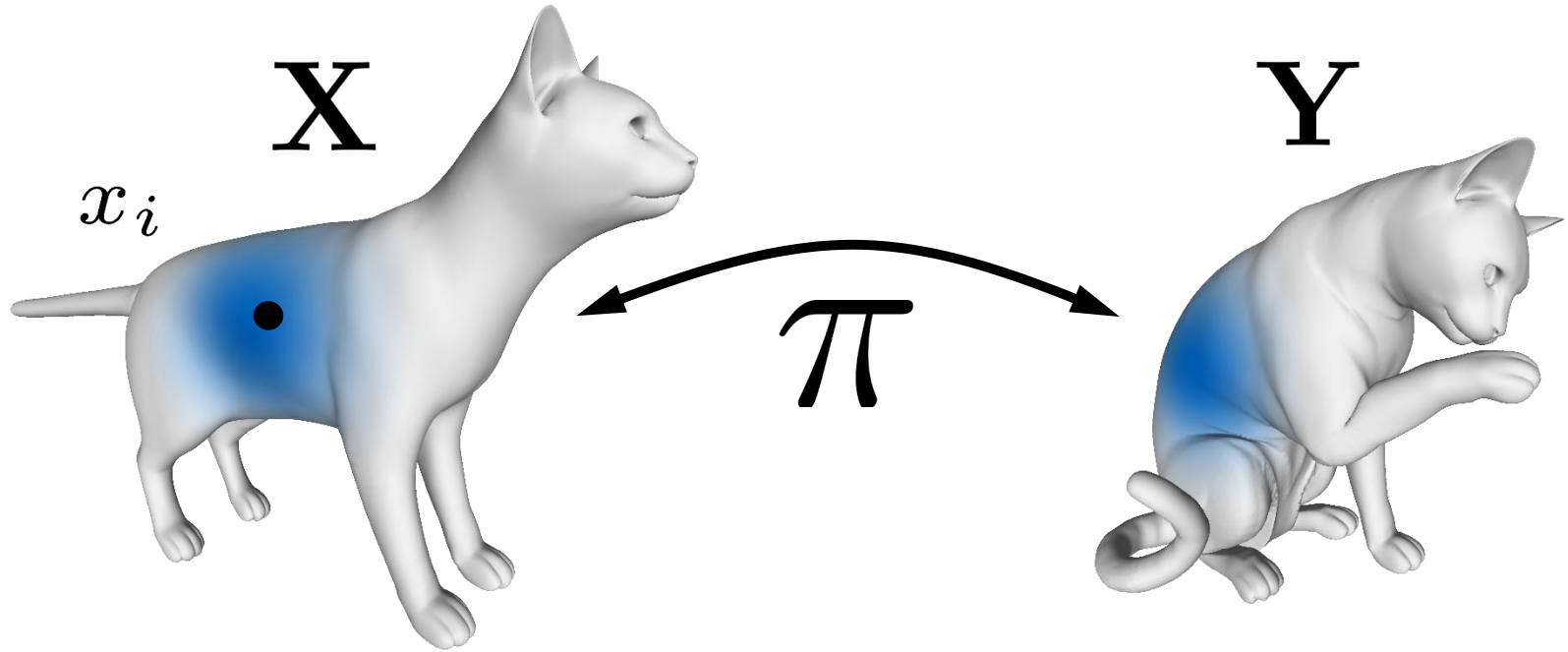
“Neighborhood maps to neighborhood”



# Cost Function

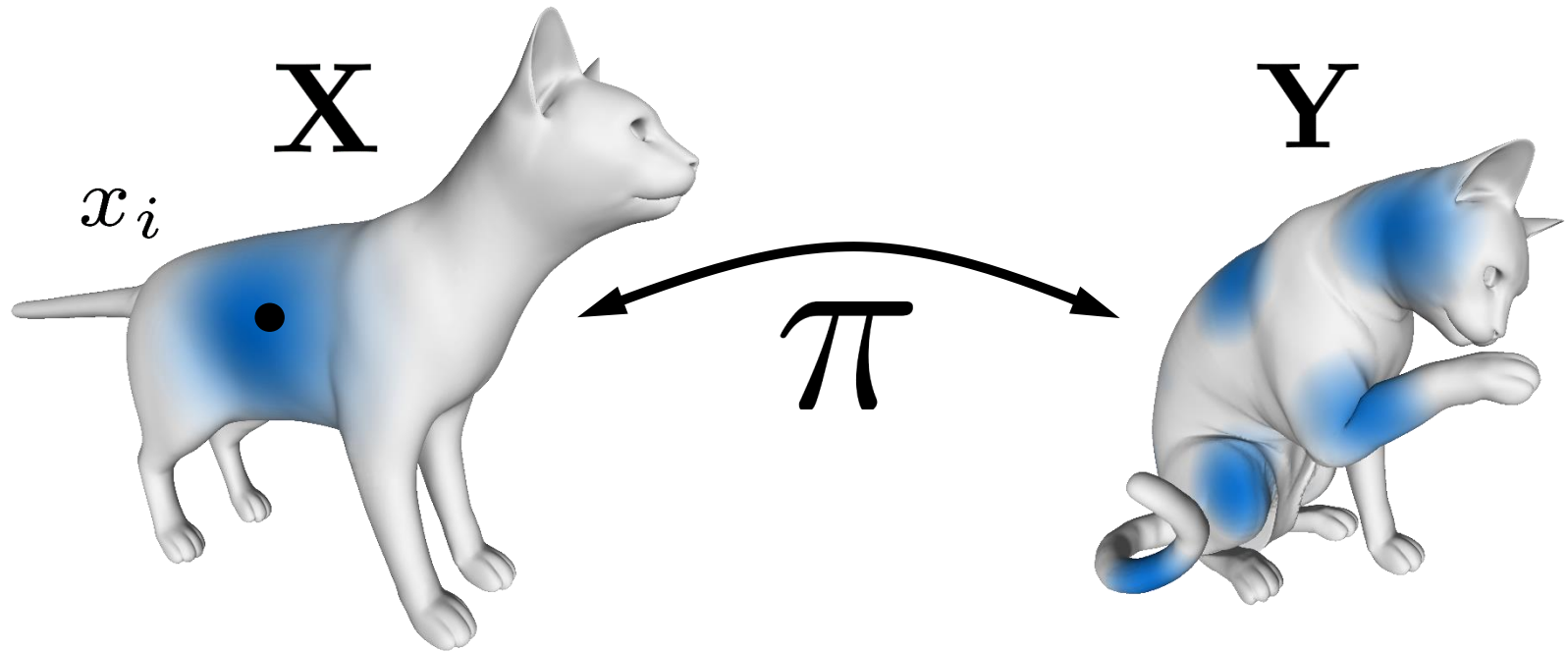


# Cost Function



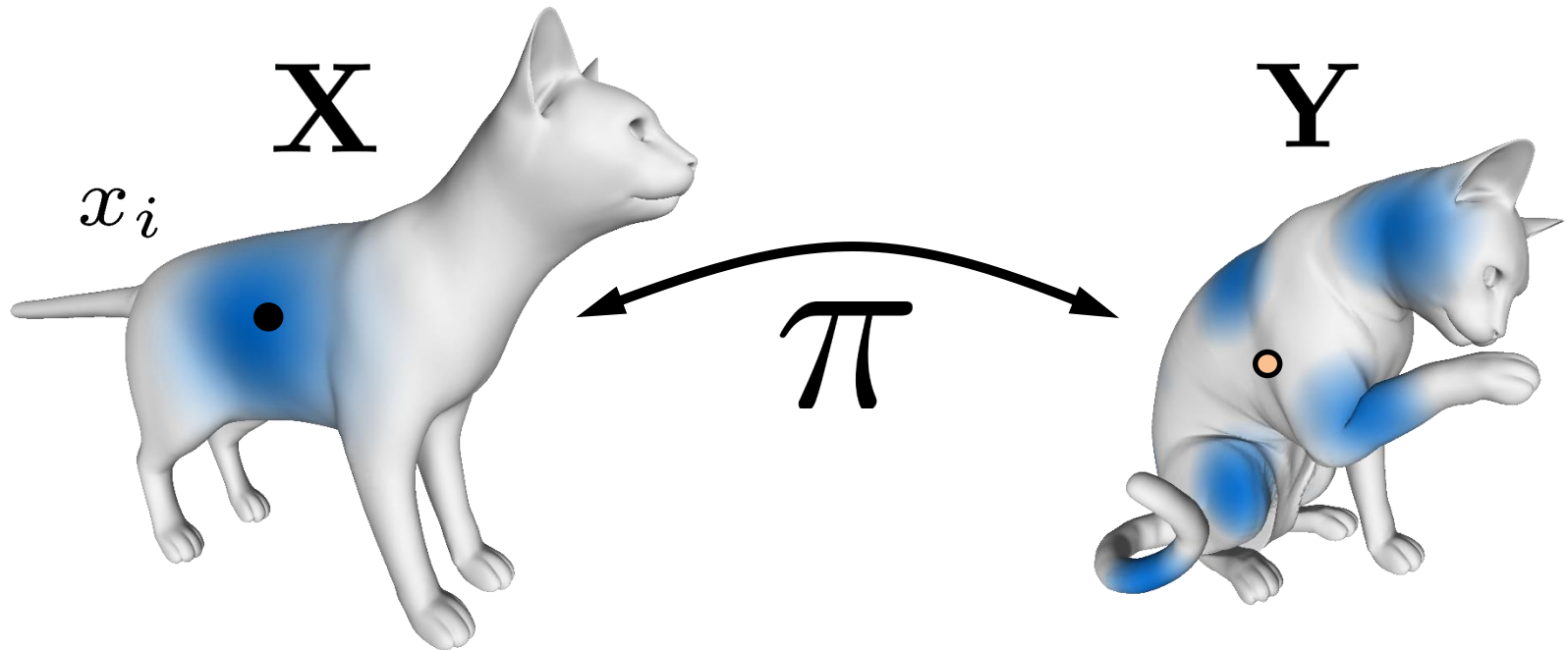
$$\mathcal{C}(\pi) = \sum_{\mathbf{X}} \text{var}(\pi(W_{x_i} \mu))$$

# Cost Function



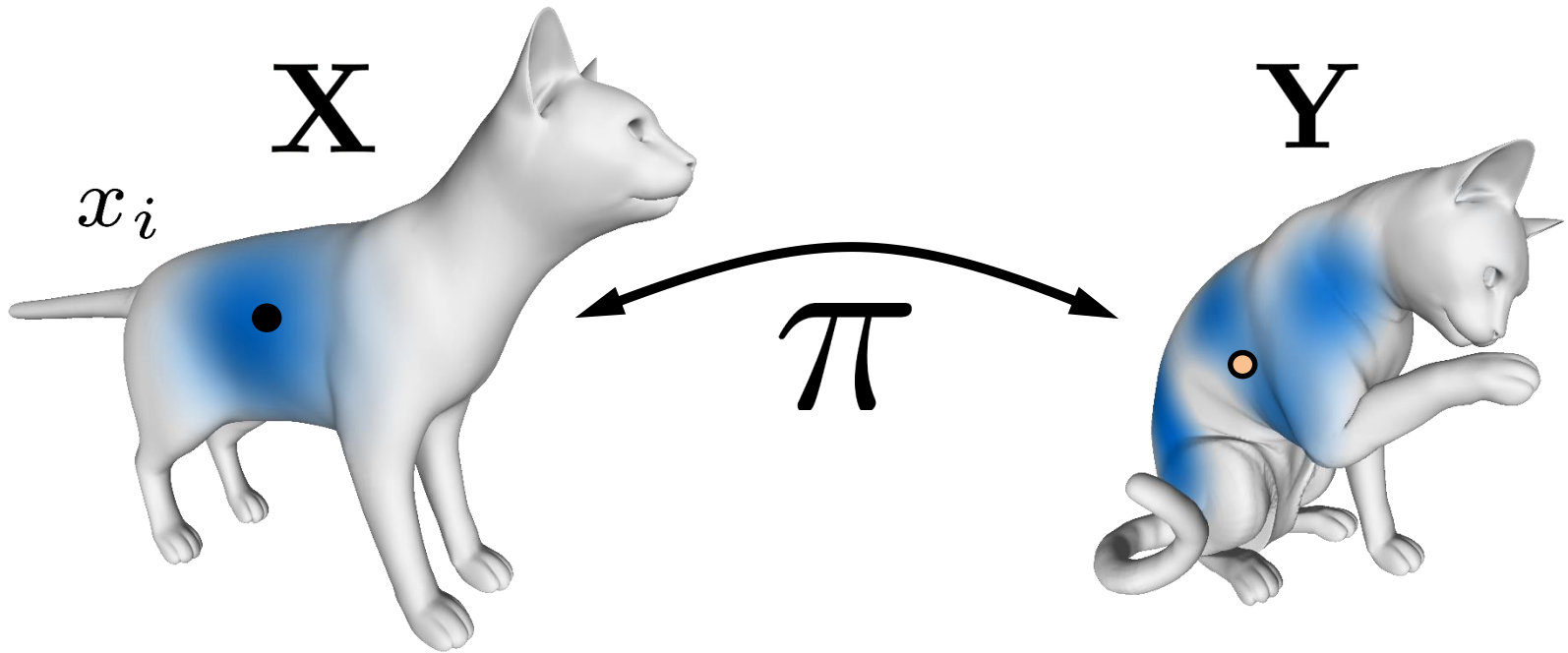
$$\mathcal{C}(\pi) = \sum_{\mathbf{X}} \text{var}(\pi(W_{x_i} \mu))$$

# Cost Function



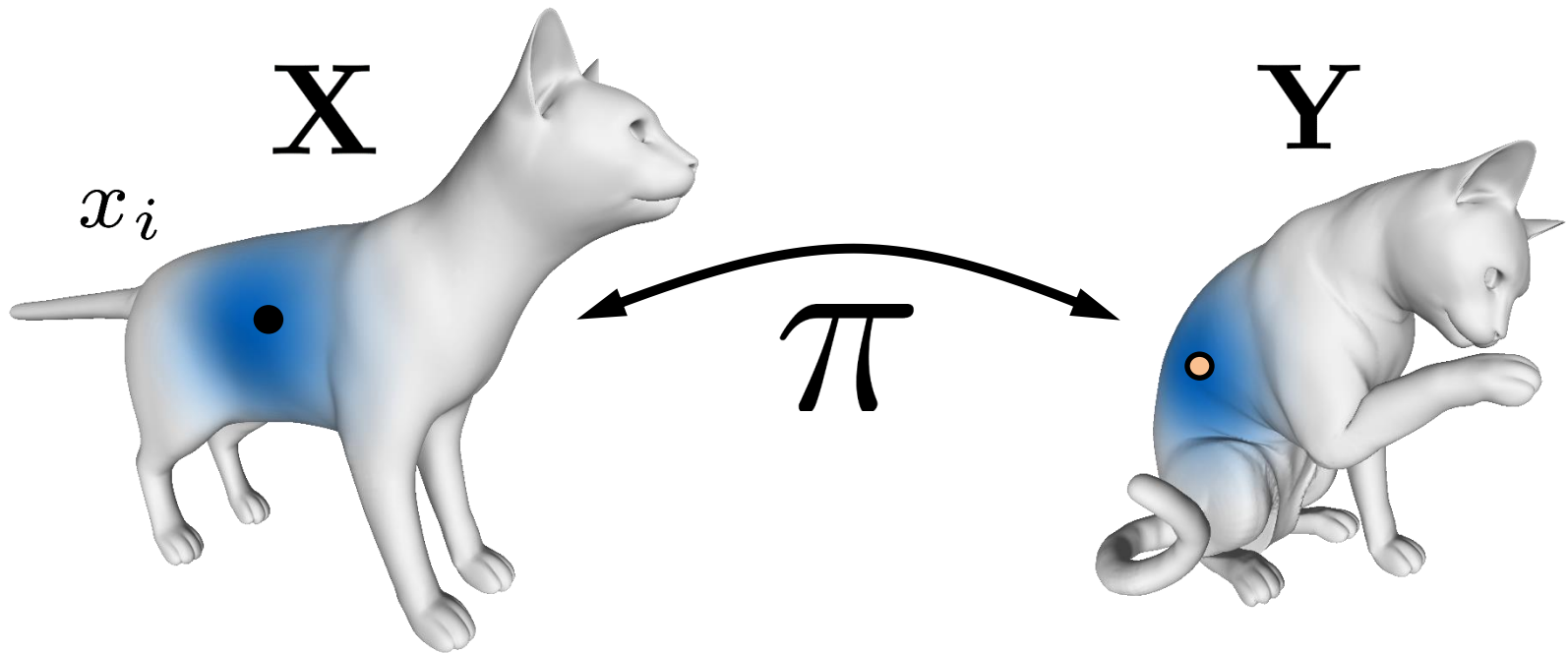
$$\mathcal{C}(\pi, \eta) = \sum_{\mathbf{X}} \text{var}(\pi(W_{x_i} \mu), \eta_{x_i})$$

# Cost Function



$$\mathcal{C}(\pi, \eta) = \sum_{\mathbf{X}} \text{var}(\pi(W_{x_i} \mu), \eta_{x_i})$$

# Cost Function



$$\mathcal{C}(\pi, \eta) = \sum_{\mathbf{X}} \text{var}(\pi(W_{x_i} \mu), \eta_{x_i})$$



# Minimization

$$E(\pi) = \int_{\mathbf{X}} \text{var } \pi_{\mathbf{X}} \left( \frac{W_x \mu}{\text{mass}(W_x \mu)} \right) d\mu(x) \\ + \int_{\mathbf{Y}} \text{var } \pi_{\mathbf{Y}} \left( \frac{W_y \nu}{\text{mass}(W_y \nu)} \right) d\nu(y)$$

Non-convex!

**Reformulation with auxiliary variables.** Given a measure  $\mu = \sum \mu_i \delta_{x_i}$  and a point  $x$ , we denote by  $\text{var}(\mu, x)$  its variance with respect to  $x$ :

$$\text{var}(\mu, x) = \sum \mu_i d(x_i, x)^2, \quad (2)$$

geodesic barycenter

---

## Algorithm 1 Map optimization through alternating minimization

---

- 1: **function** ALTERNATING MINIMIZATION
  - 2:     **for**  $i = 1, 2, 3 \dots$  **do**
  - 3:          $\eta \leftarrow \min \mathcal{C}(\pi, \cdot)$  // move centers to geodesic barycenters
  - 4:          $\pi \leftarrow \min \mathcal{C}(\cdot, \eta)$  // solve optimal transport problem
  - 5:     **return**  $\pi$              ▷ variance-minimizing transport plan
-

# Minimization

$$E(\pi) = \int_{\mathbf{X}} \text{var } \pi_{\mathbf{X}} \left( \frac{W_x \mu}{\text{mass}(W_x \mu)} \right) d\mu(x) + \int_{\mathbf{Y}} \text{var } \pi_{\mathbf{Y}} \left( \frac{W_y \nu}{\text{mass}(W_y \nu)} \right) d\nu(y)$$

Non-convex!

**Reformulation with auxiliary variables.** Given a measure  $\mu = \sum \mu_i \delta_{x_i}$  and a point  $x$ , we denote by  $\text{var}(\mu, x)$  its variance with respect to  $x$ :

$$\text{var}(\mu, x) = \sum \mu_i d(x_i, x)^2, \quad (2)$$

geodesic barycenter

in diffusion space

---

## Algorithm 1 Map optimization through alternating minimization

---

- 1: **function** ALTERNATING MINIMIZATION
  - 2:     **for**  $i = 1, 2, 3 \dots$  **do**
  - 3:          $\eta \leftarrow \min \mathcal{C}(\pi, \cdot)$  // move centers to geodesic barycenters
  - 4:          $\pi \leftarrow \min \mathcal{C}(\cdot, \eta)$  // solve optimal transport problem
  - 5:     **return**  $\pi$              ▷ variance-minimizing transport plan
- 

Sinkhorn iteration

[Cuturi et al.]



# Minimization

$$E(\pi) = \int_{\mathbf{X}} \text{var } \pi_{\mathbf{X}} \left( \frac{W_x \mu}{\text{mass}(W_x \mu)} \right) d\mu(x) + \int_{\mathbf{Y}} \text{var } \pi_{\mathbf{Y}} \left( \frac{W_y \nu}{\text{mass}(W_y \nu)} \right) d\nu(y)$$

Non-convex!

**Reformulation with auxiliary variables.** Given a measure  $\mu = \sum \mu_i \delta_{x_i}$  and a point  $x$ , we denote by  $\text{var}(\mu, x)$  its variance with respect to  $x$ :

$$\text{var}(\mu, x) = \sum \mu_i d(x_i, x)^2, \quad (2)$$

geodesic barycenter

Coarse to fine

---

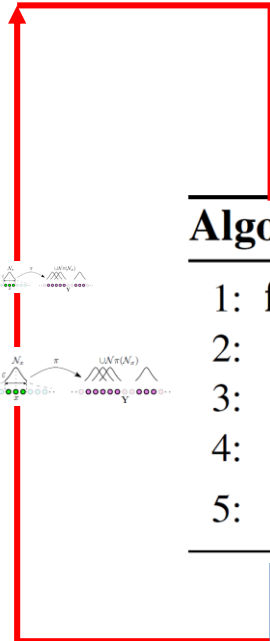
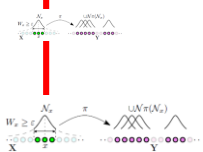
## Algorithm 1 Map optimization through alternating minimization

---

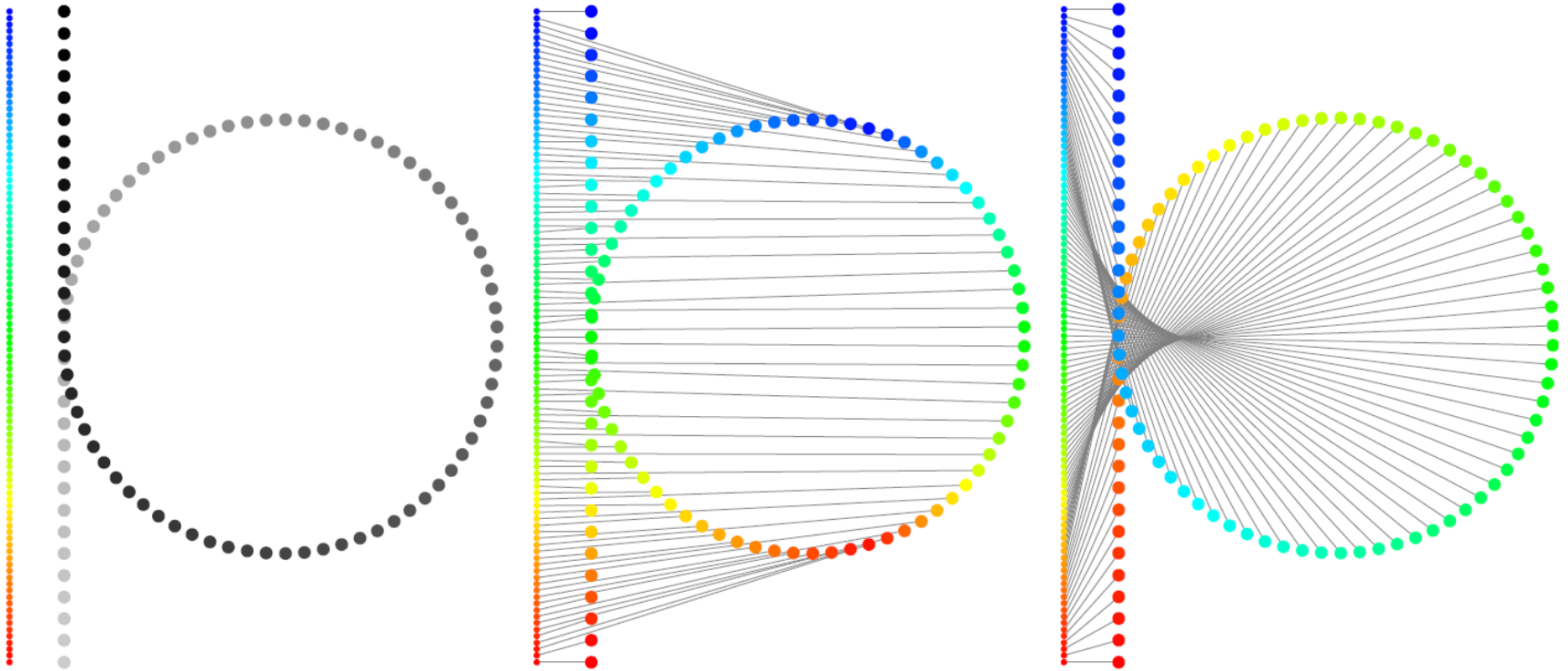
- 1: **function** ALTERNATING MINIMIZATION
  - 2:   **for**  $i = 1, 2, 3 \dots$  **do**
  - 3:      $\eta \leftarrow \min \mathcal{C}(\pi, \cdot)$  // move centers to geodesic barycenters
  - 4:      $\pi \leftarrow \min \mathcal{C}(\cdot, \eta)$  // solve optimal transport problem
  - 5:   **return**  $\pi$                      $\triangleright$  variance-minimizing transport plan
- 

in diffusion space

Sinkhorn iteration



# Minimization



Input

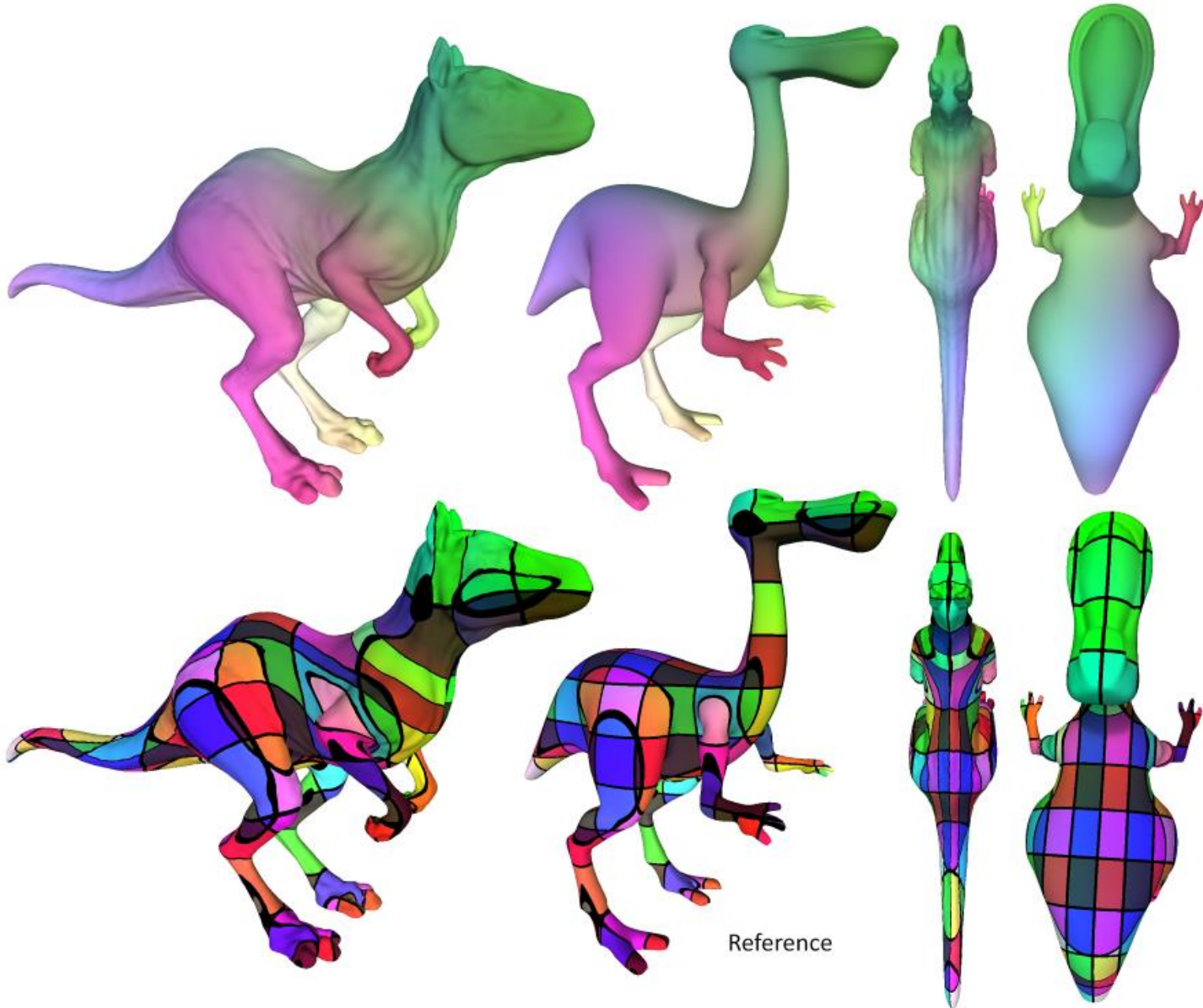
Initial map  
(nearest)

EM iterations

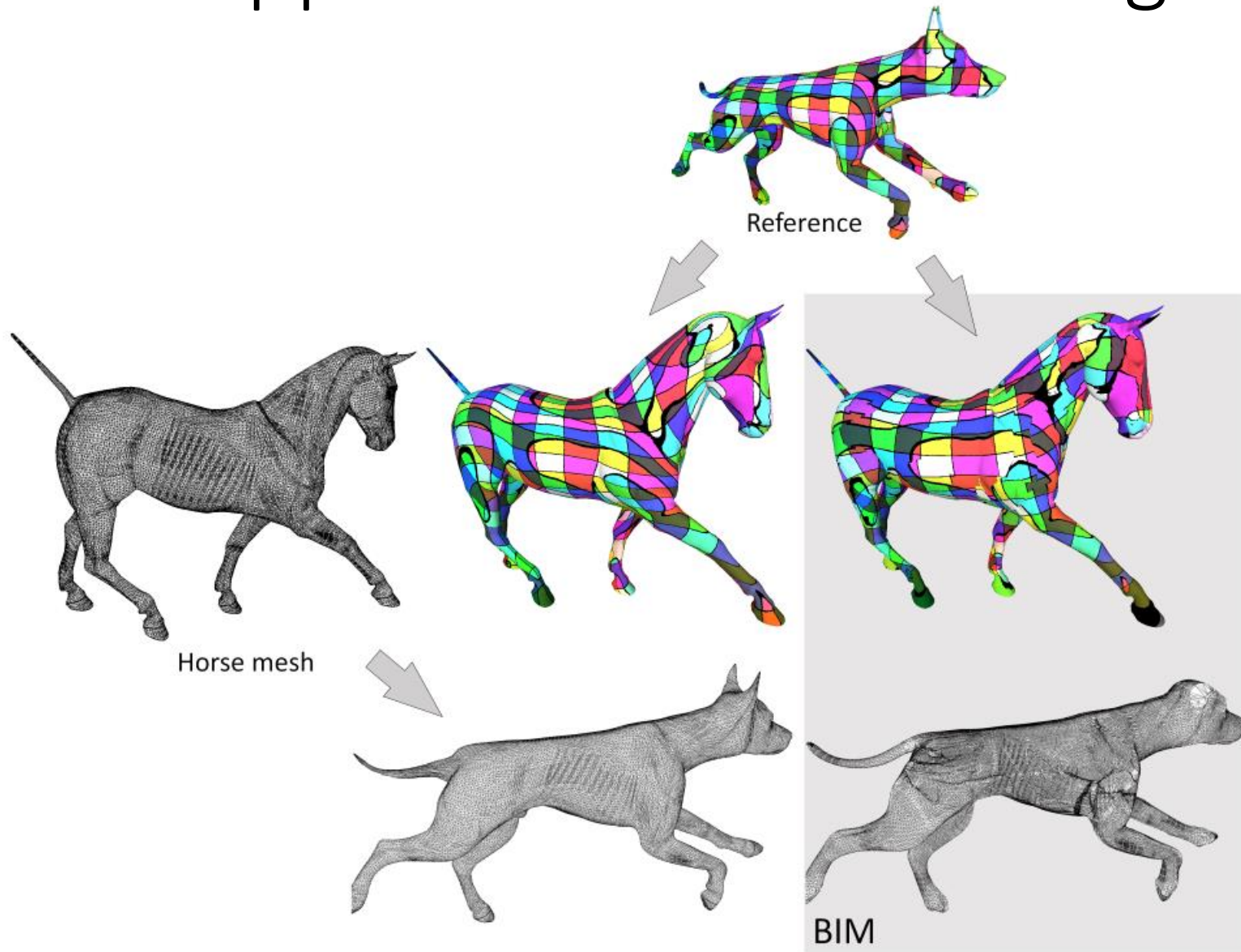
# Isometric case

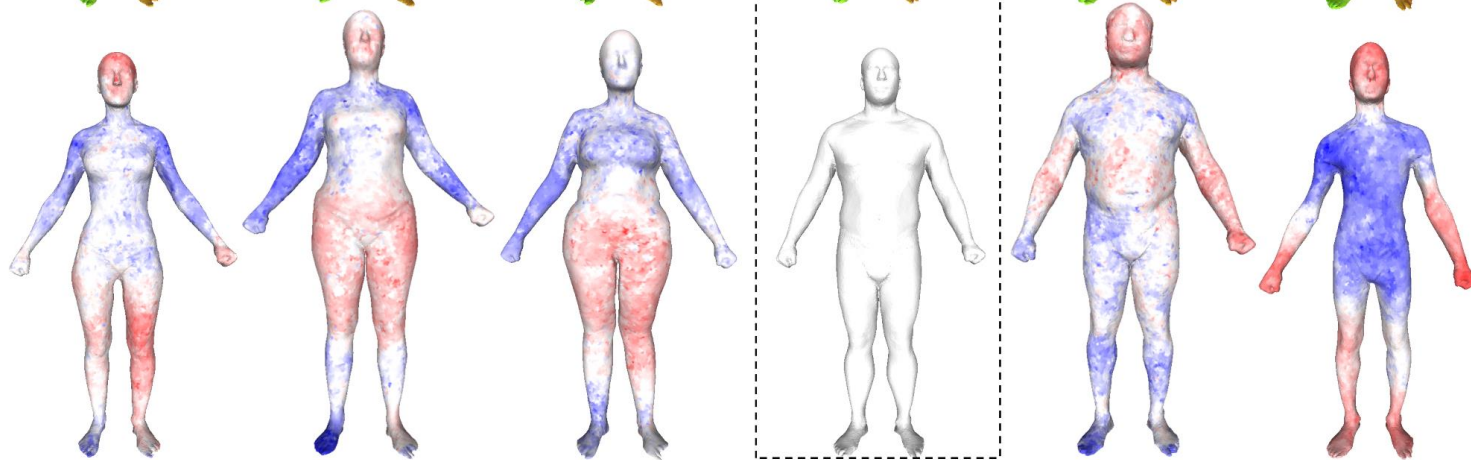
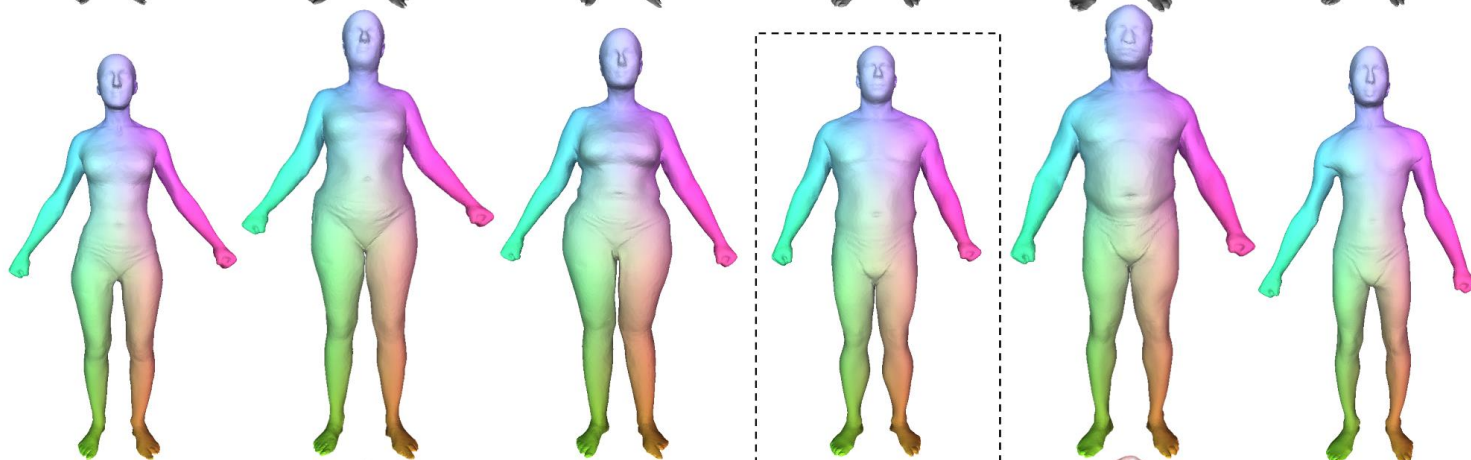


# Non-isometric



# Application to Remeshing



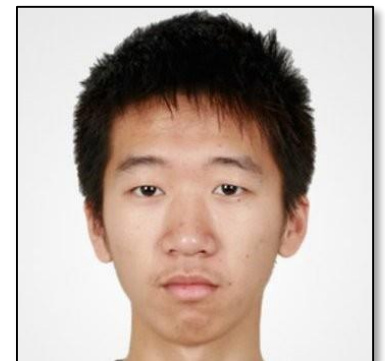


# Resilience to Topological Noise



# Higher-order Meshes

Joint work with **Leman Feng**, Laurent Busé,  
Hervé Delingette and Mathieu Desbrun



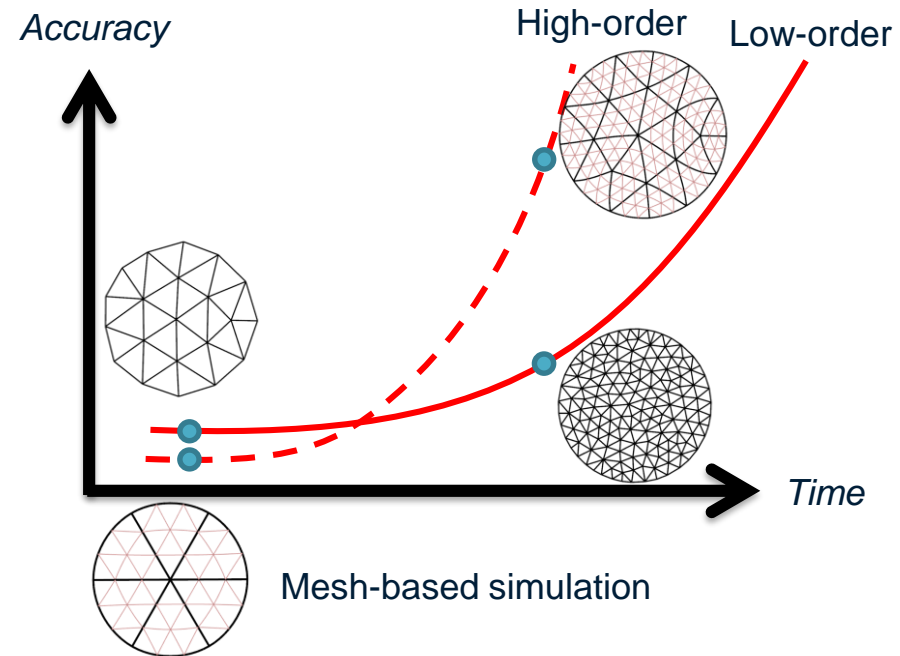


# Motivations

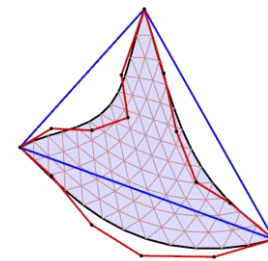
Simulation of laparoscopic liver surgery

Meshing = **essential preprocessing step**

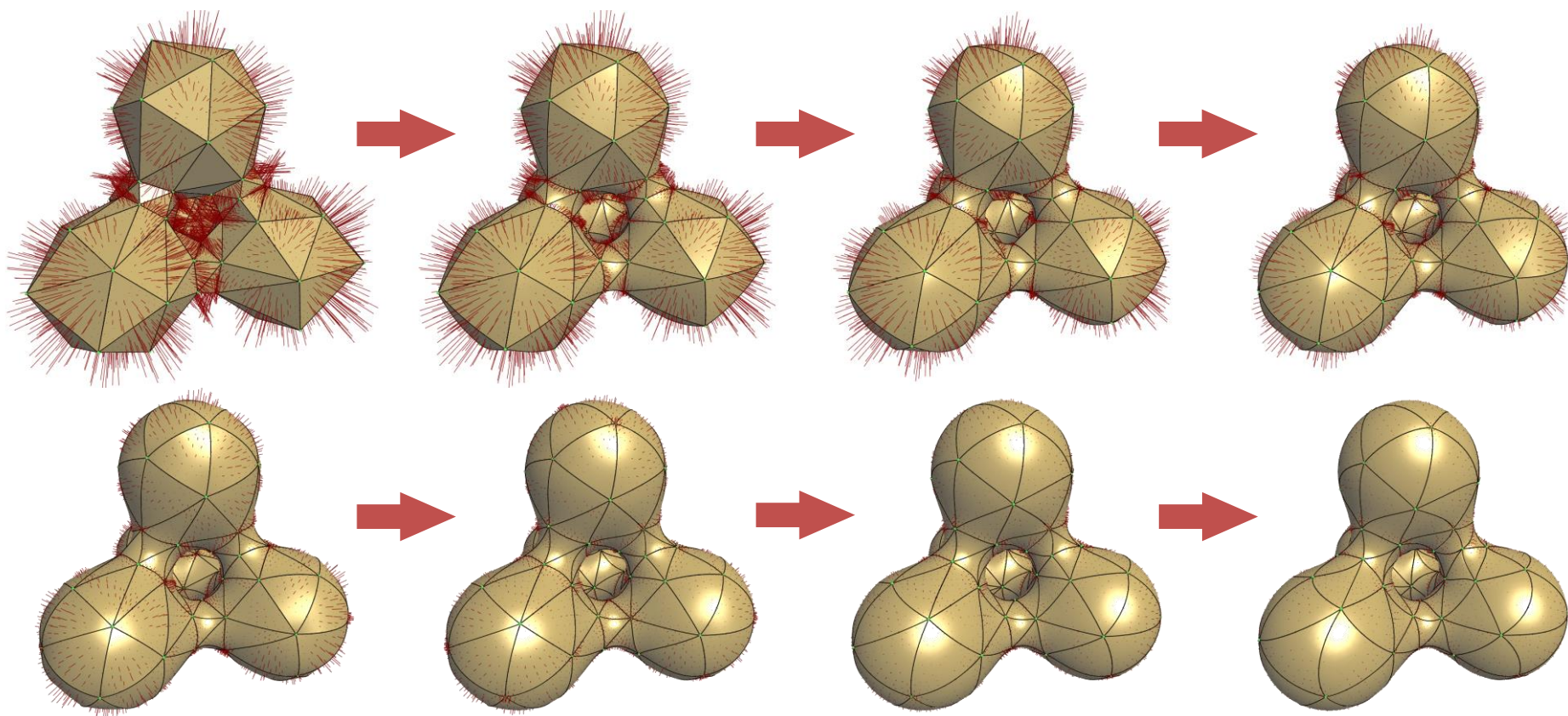
- Curved domains and complex phenomena motivate **higher-order elements** & basis functions [Roth '98, Weber '11, Suwelack '13, Geuzaine '15]
- Smaller element count.



# Higher-order Meshing



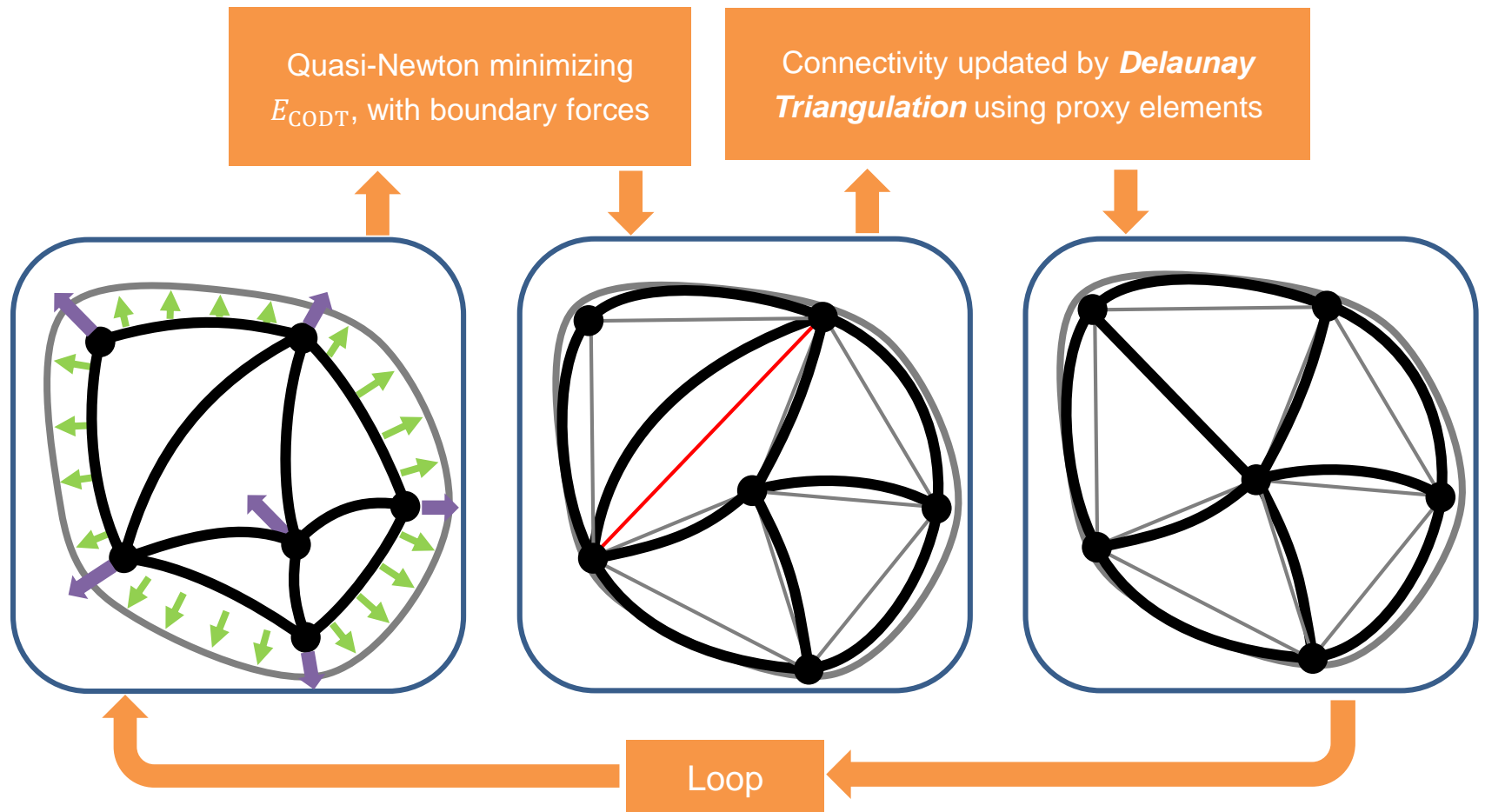
Bézier element



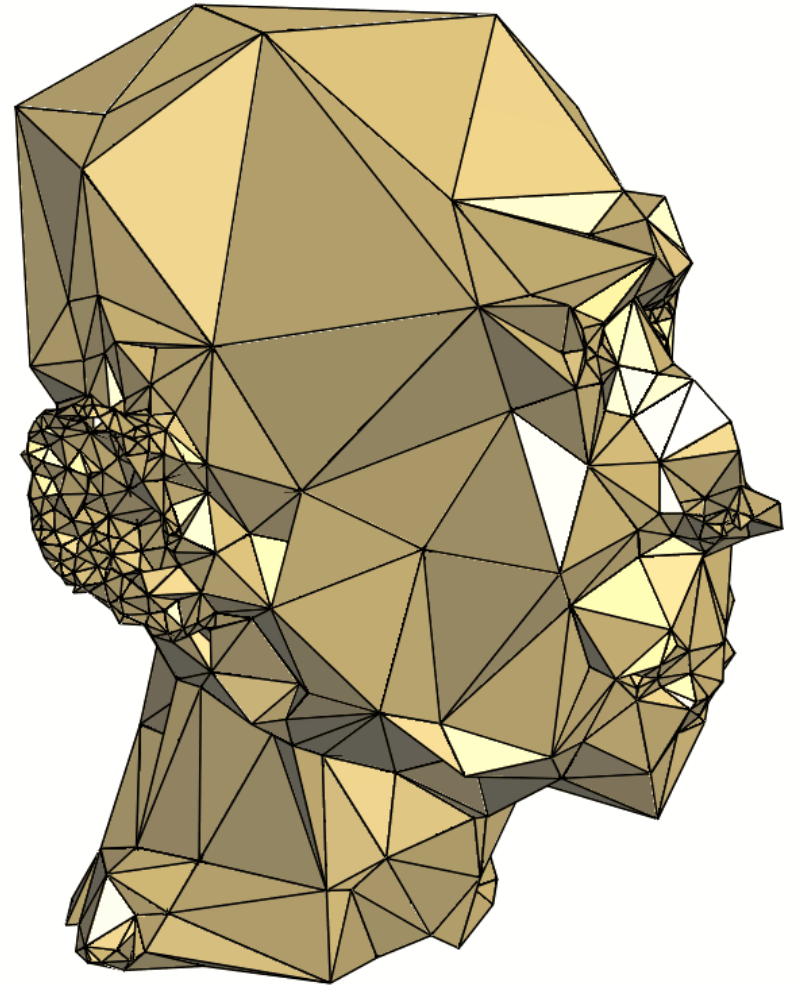
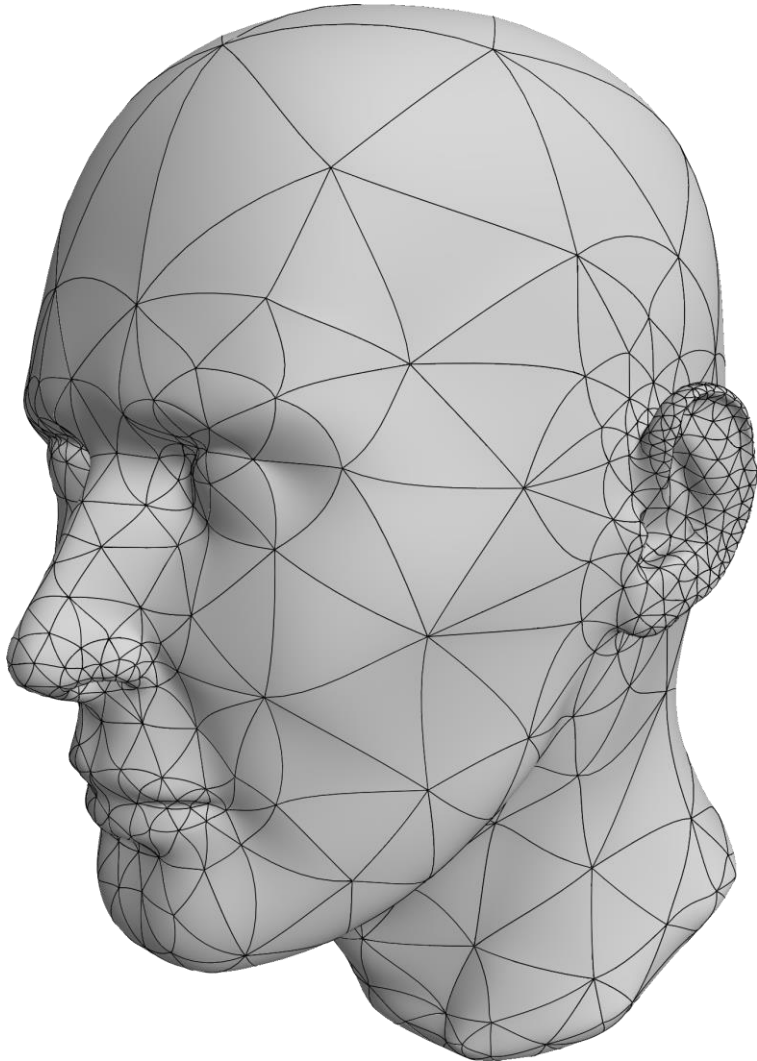
Approximation errors (magnified)

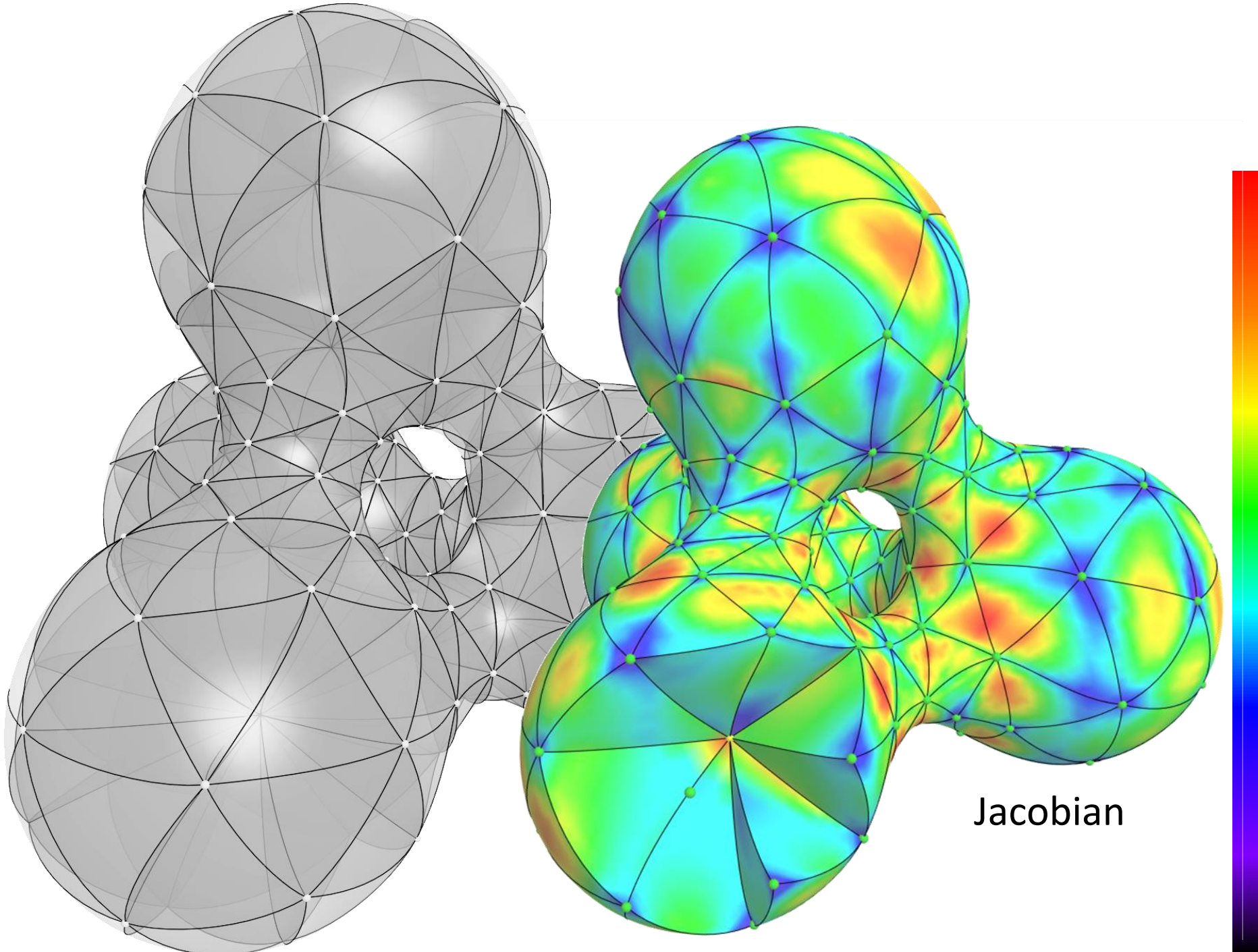
200 vertices, cubic patches

# Algorithm



# Algorithm

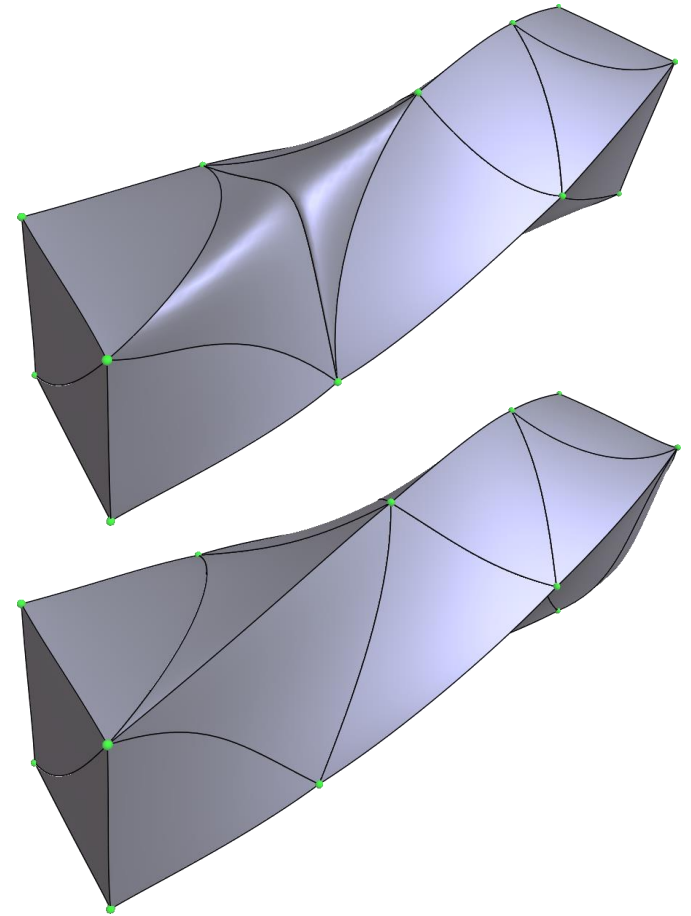




Jacobian

# On-going Work

- Sharp features & boundaries
- H/P dilemma
  - Order vs element size
  - Learn prediction



Toolbox

# Methodological Keywords

## Computational geometry

- Delaunay-Voronoi
- Kinetic data structures

## Geometric computing



## Applied mathematics

- Variational formulations
- Optimal transportation

## Stochastic geometry

- Spatial point processes
- Monte Carlo sampling

## Machine learning

- Support vector machines
- Random forests
- Deep neural networks
- ...



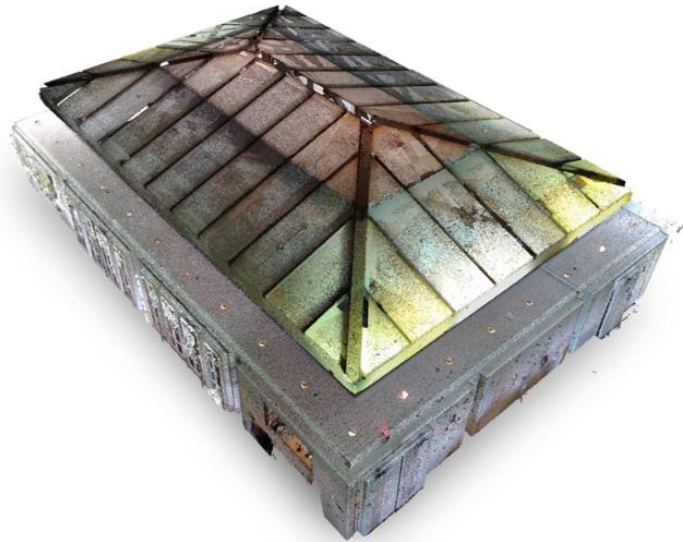


# Recent and Outlook

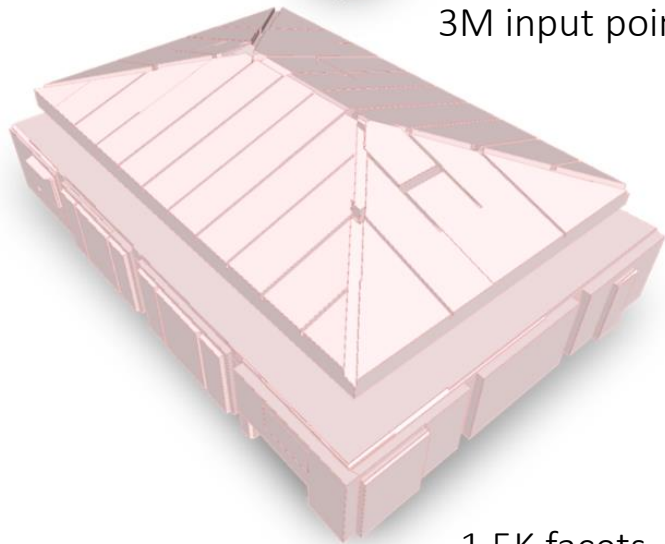
Cognitive 3D models



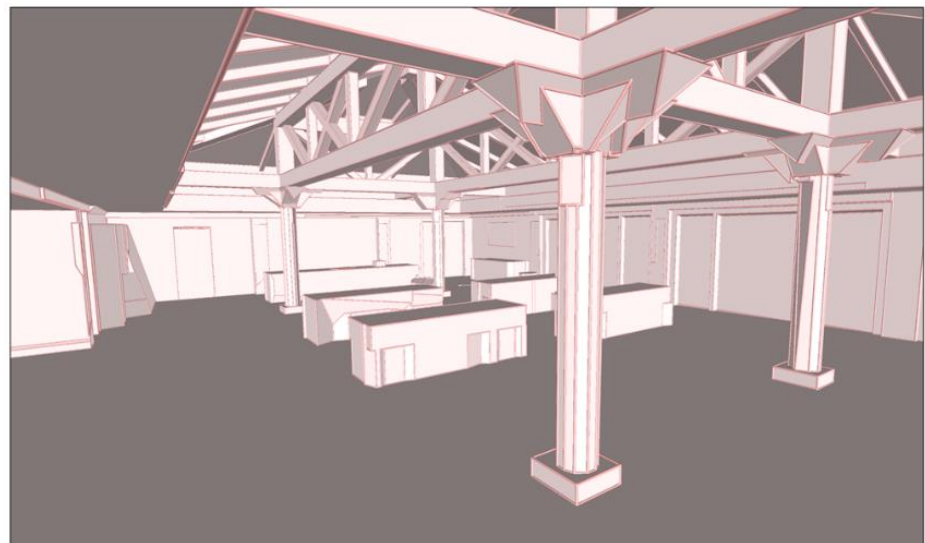
# 3D Reconstruction...



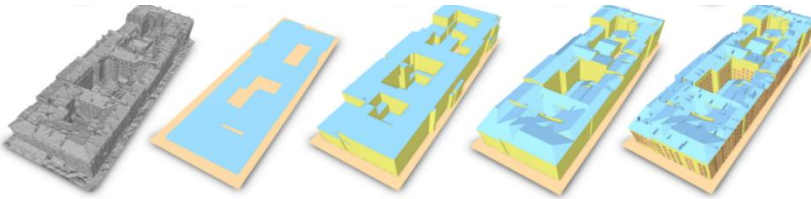
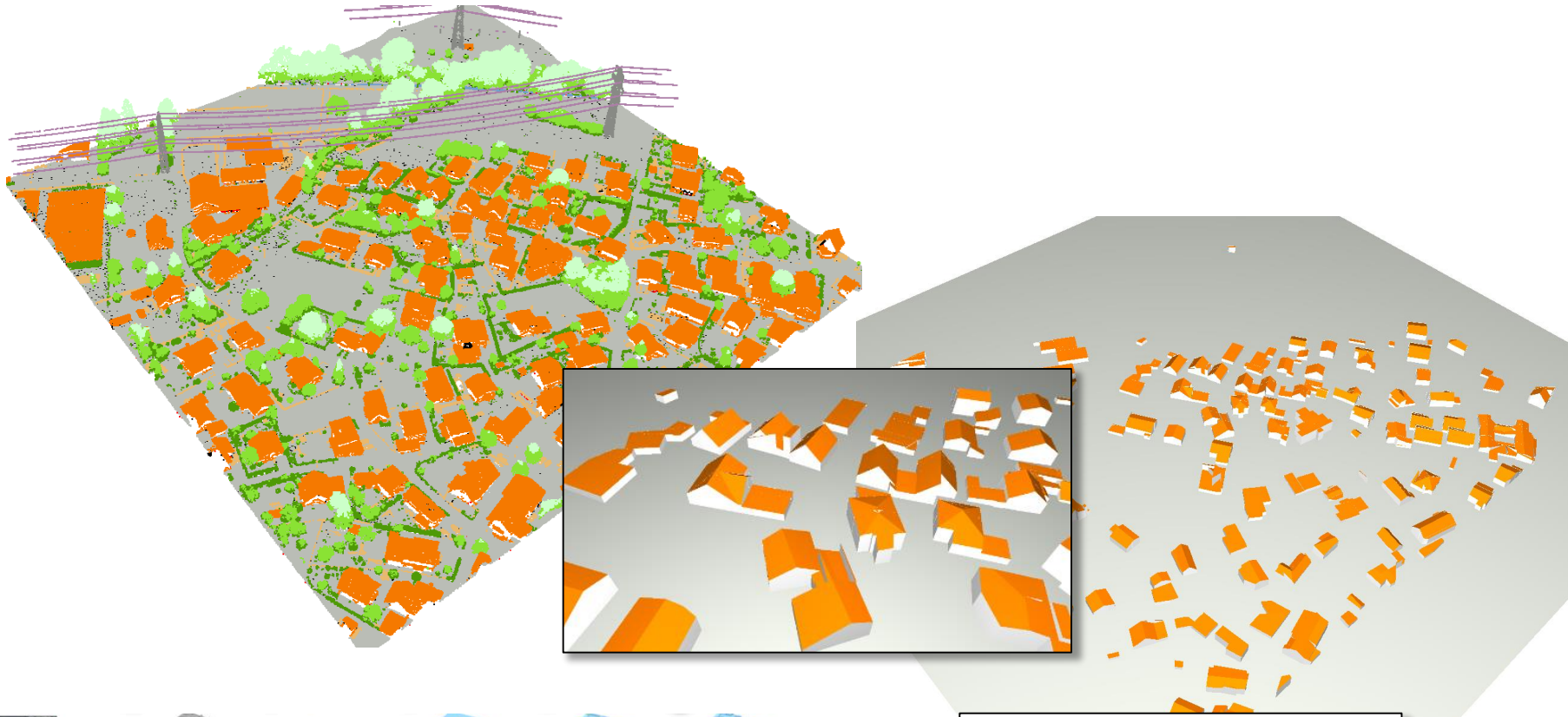
3M input points



1.5K facets



# ...and Support of Domain Knowledge



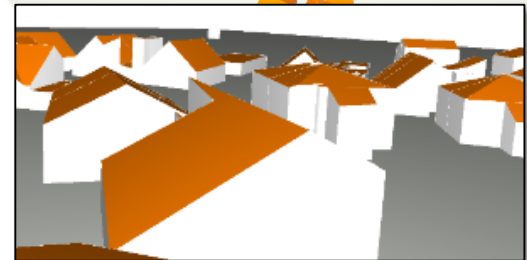
CityGML Level of detail

0

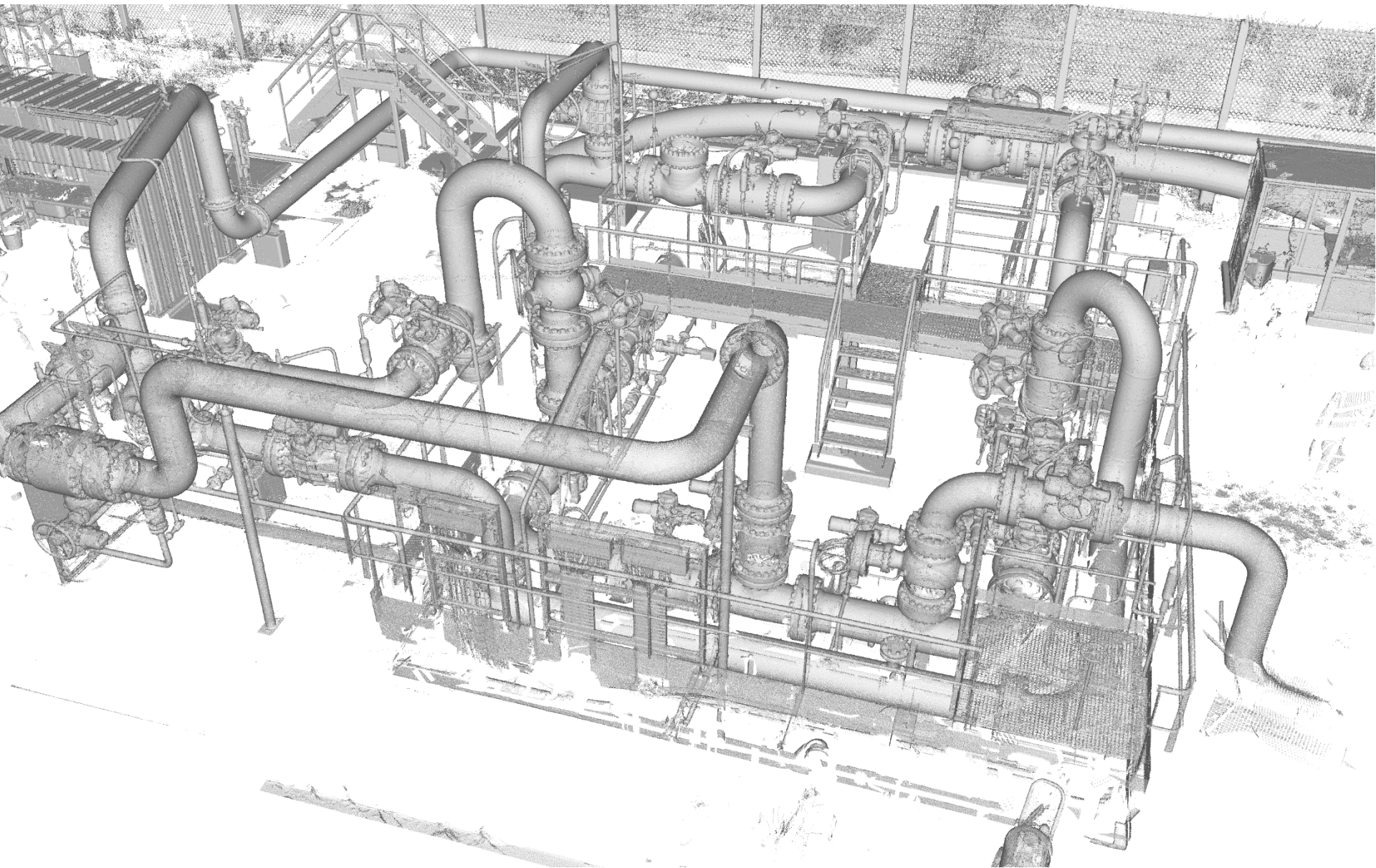
1

2

3



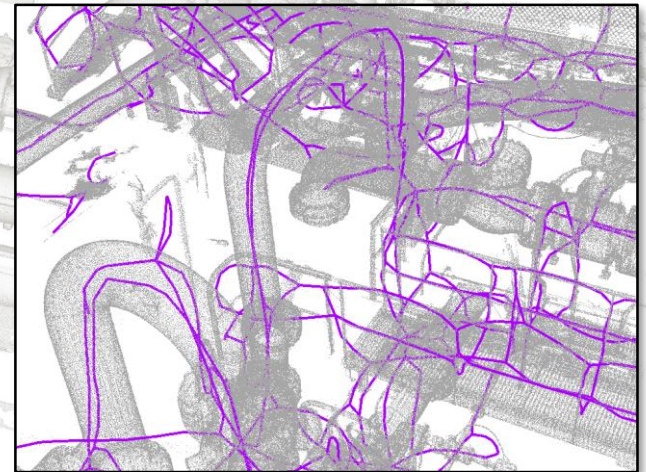
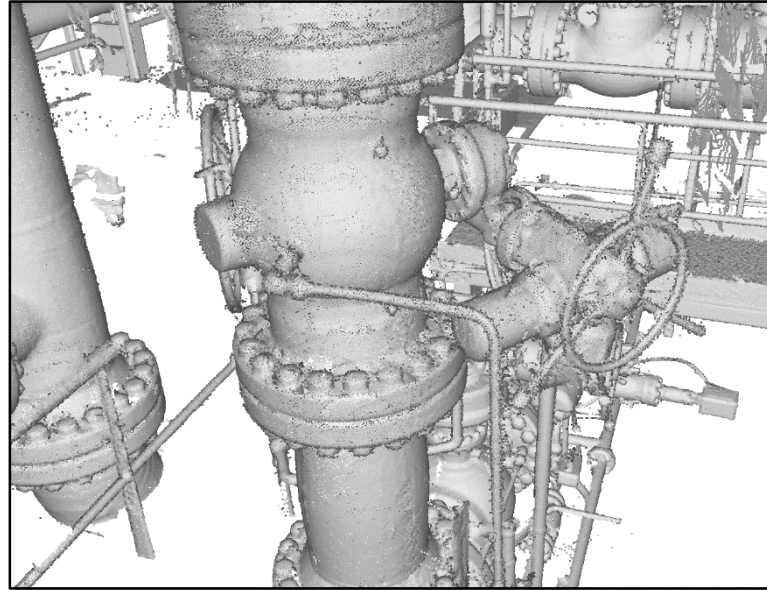
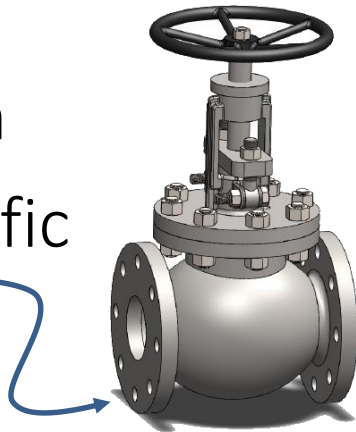
# Raw Point Cloud...



# ...and Cognitive 3D Model

## Cognitive:

- Searchable
- Insights from digital realities
  - Abstraction
  - Detection
  - Segmentation
  - Domain-specific queries



Pipe-run



# Recent and Outlook

Physics-informed modeling

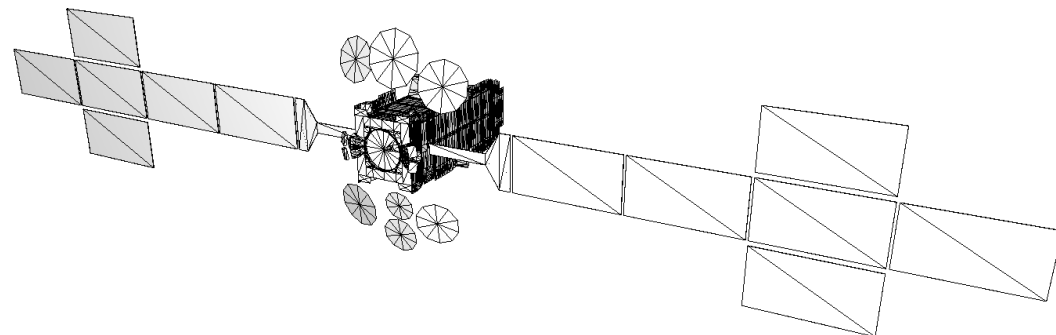
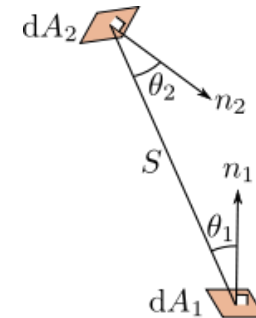
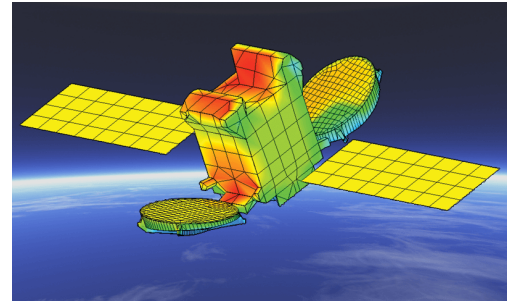
# Radiative Thermal Simulation



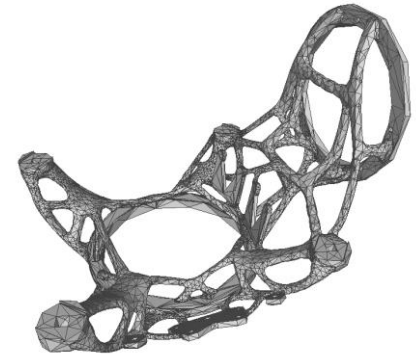
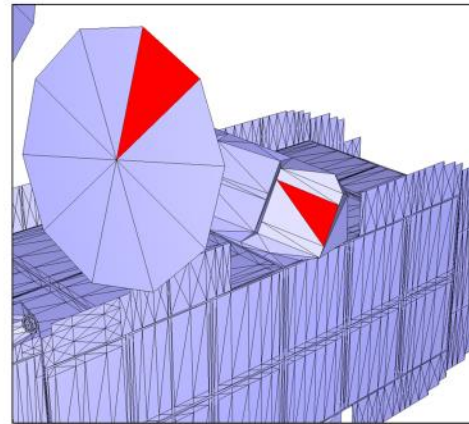
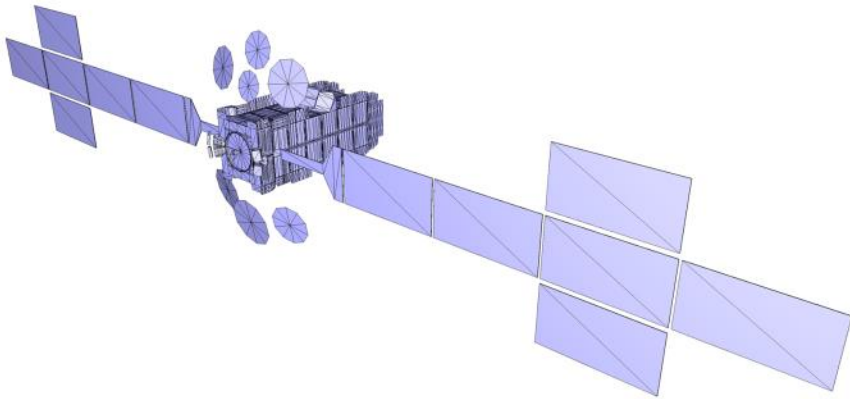
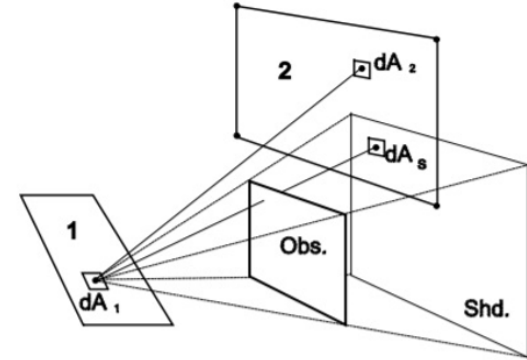
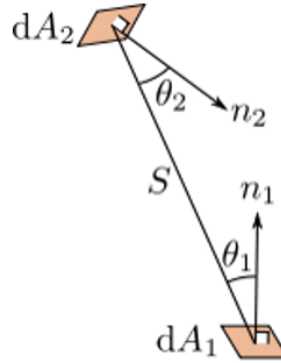
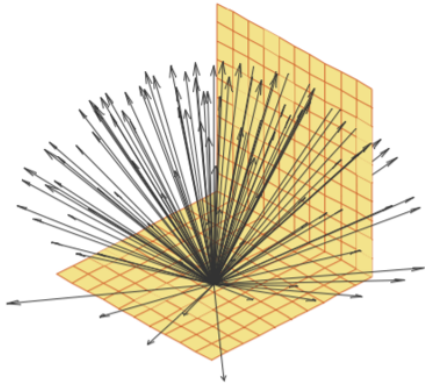
Vincent  
Vadez

## Goals

- View factors
- Thermal-aware geometric approximation
- Trade accuracy for time
- Guarantees: error bounds
  - under wide range of configurations and conditions

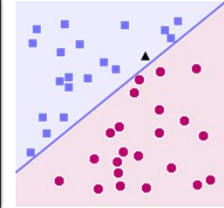
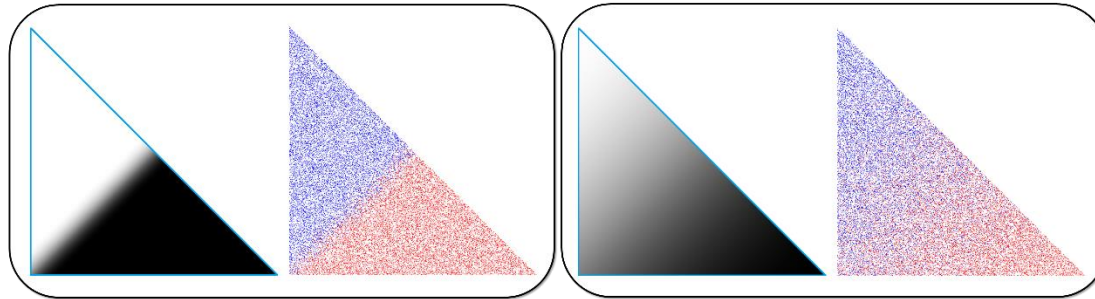
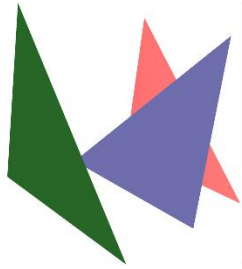


# Geometric View Factors

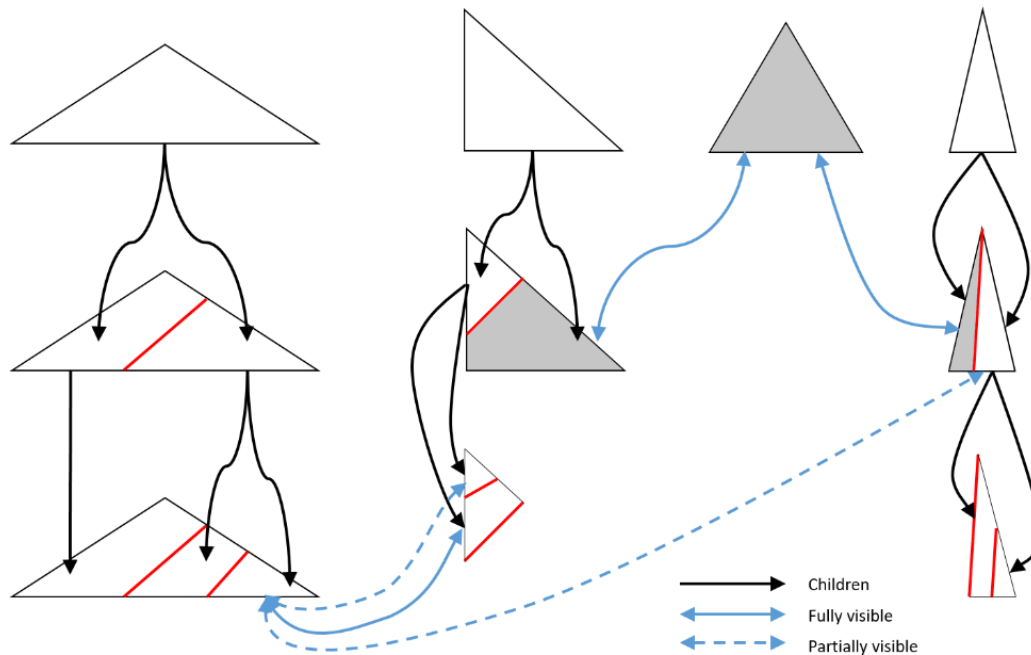




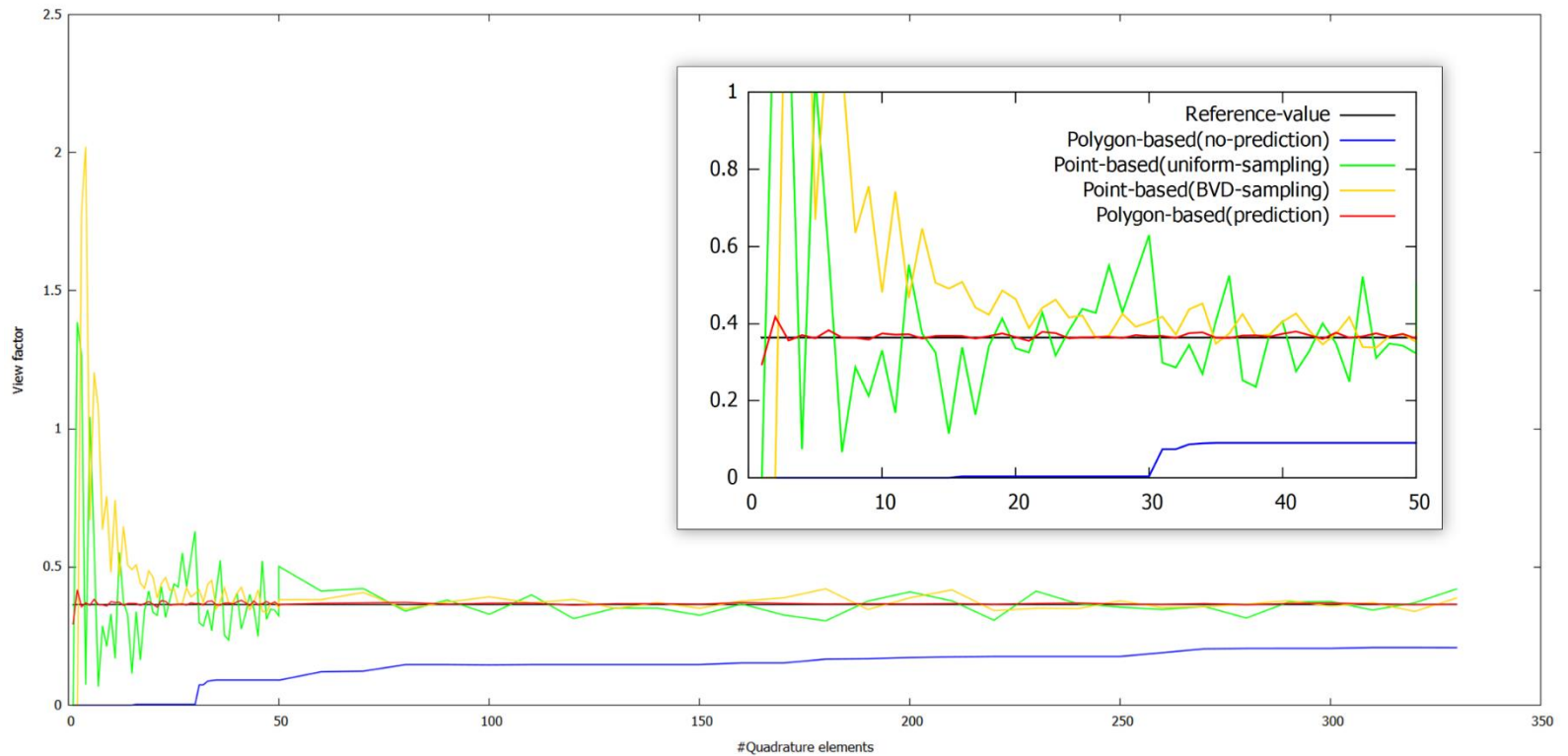
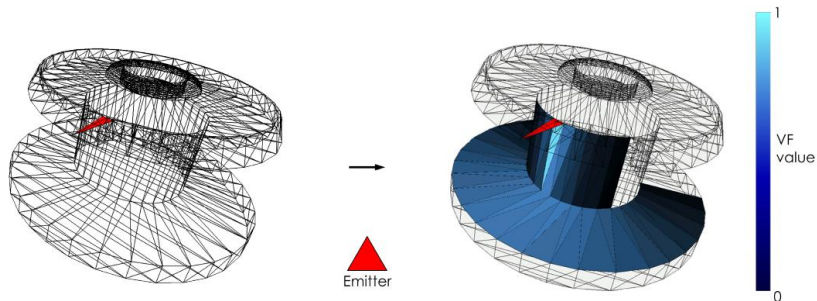
# Progressive View Factors



SVM

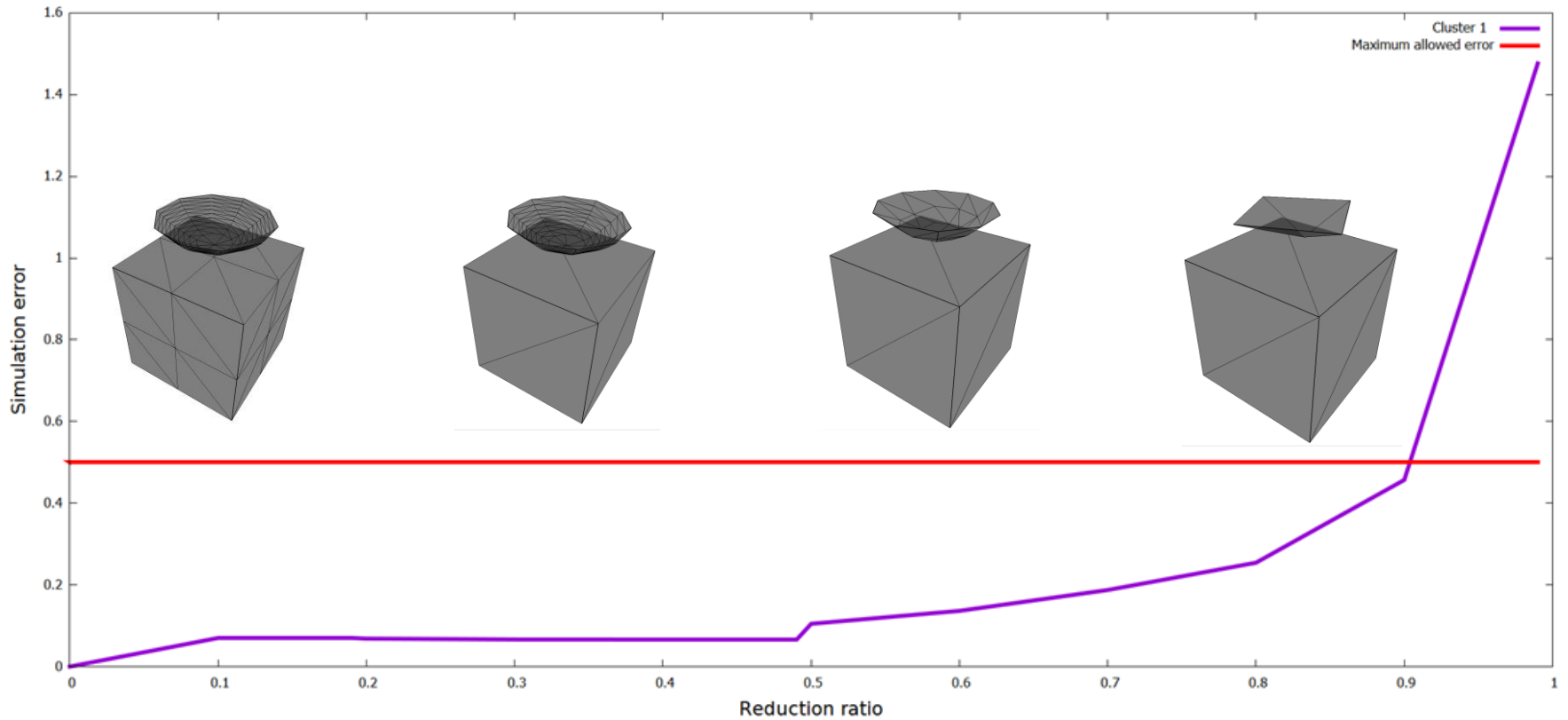


# Predicting Geometric View Factors

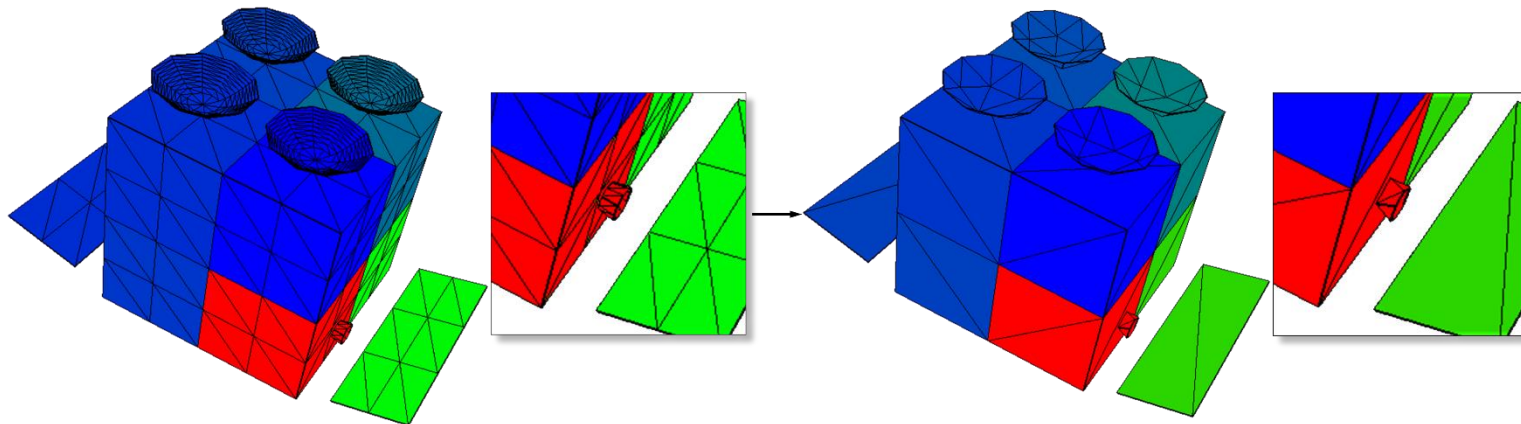
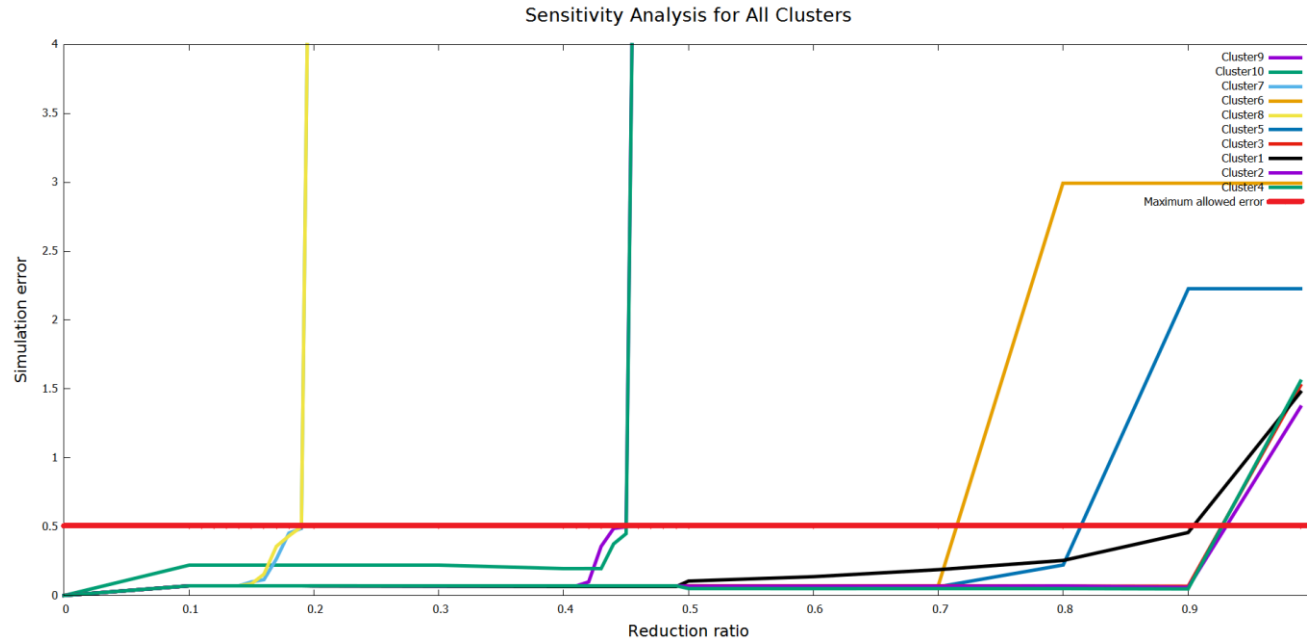


# Physics-informed Mesh Reduction

Sensitivity Analysis for One Cluster



# Physics-informed Mesh Reduction

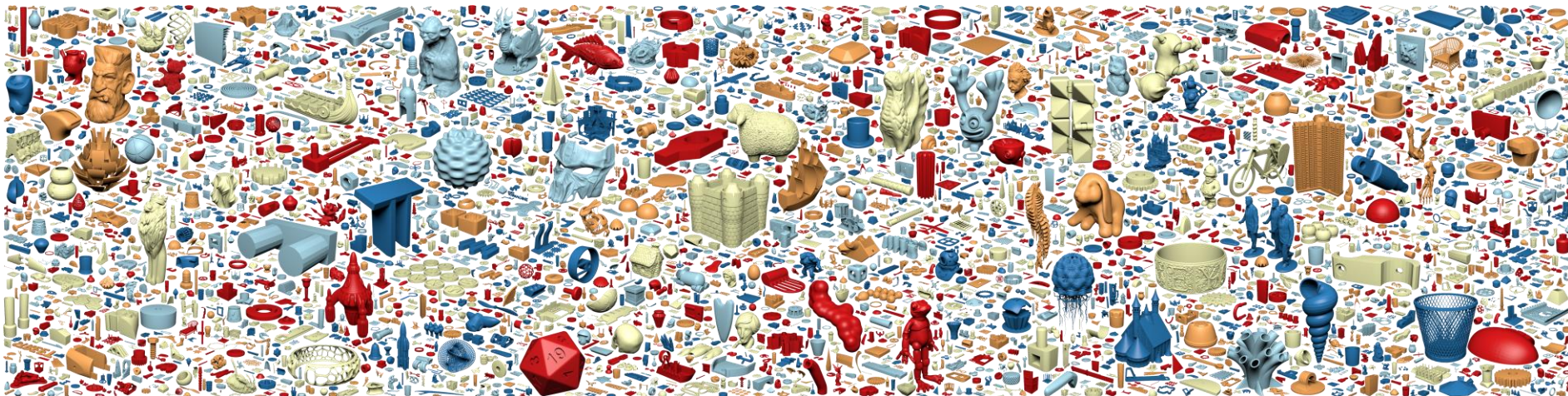


# On-Going Work

- Learn physics-informed error metric
- Physics-informed 3D reconstruction

# Physics-informed Modeling

- Improve discretizations
  - Simulation = discrete differential operators
  - Operator-specific mesh optimization



Training data

# Thank you.

Funding

*Inria*



Consolidator Grant “IRON”  
Robust Geometry Processing  
Proof of Concept “TITANIUM”

**3iA** Côte d’Azur

Institut interdisciplinaire  
d’intelligence artificielle

